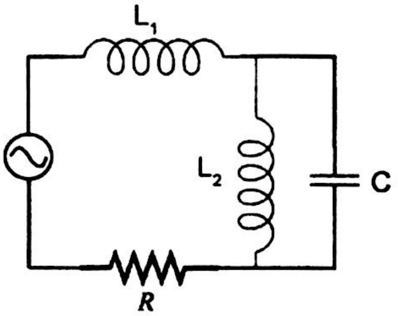
Name: 1C Midterm Review - Week 8

1. **Previous Midterm** *(by Prof. Simon).* The circuit below has an AC voltage source, where $V = V_0 cos(\omega t)$.



$$\tilde{X}_L = i\omega L$$

$$\tilde{X}_C = \frac{-i}{\omega C}$$

(a) What is the circuit's total impedance, magnitude and phase angle? [**8 points**]

$$\tilde{Z} = R + i\omega L_1 + \frac{1}{\frac{1}{\tilde{X}_L} + \frac{1}{\tilde{X}_C}} = R + i\omega L_1 + \frac{1}{\frac{1}{i\omega L_2} - \frac{\omega C}{i}}$$

$$= R + i\left[\omega L_1 + \frac{\omega L_2}{1-\omega^2 L_2 C}\right]$$

$$Z_0 = \left(R^2 + \left[\omega L_1 + \frac{\omega L_2}{1-\omega^2 L_2 C}\right]^2\right)^{1/2}$$

$$\phi = \arctan\left(\left[\omega L_1 + \omega L_2/(1-\omega^2 L_2 C)\right]/R\right)$$

(b) Give an expression for the current $I(t)$ in the circuit. [**8**]

$$\tilde{I}(t) = \frac{\tilde{U}}{\tilde{Z}} = \frac{V_0 e^{i\omega t}}{Z_0 e^{i\phi}} = \frac{V_0}{Z_0} e^{i\omega t - i\phi}$$

$$\Rightarrow I(t) = \frac{V_0}{Z_0}\cos(\omega t - \phi)$$

(c) What is the average power deliverd by the voltage source? [**4**]

$$\langle\tilde{P}\rangle = \langle\tilde{U}\tilde{I}\rangle = \left\langle\frac{\tilde{U}^2}{\tilde{Z}}\right\rangle = \frac{V_0^2}{Z_0}\underbrace{\langle\cos^2(\omega t)\rangle}_{1/2} e^{i\phi}$$

$$\Rightarrow \langle P\rangle = Re(\langle\tilde{P}\rangle) = \frac{V_0}{2Z_0}\cos(\phi)$$

(d) If there is a resonant frequency, what is it? [**7**]

$Im(\tilde{Z}) = 0$ possible?

$$\omega L_1 + \frac{\omega L_2}{1-\omega^2 L_2 C} = 0$$

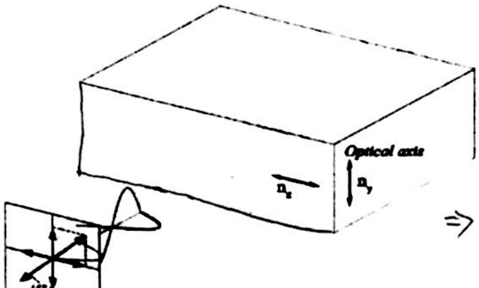
$$\Rightarrow +L_2 = -L_1(1-\omega^2 L_2 C)$$

$$\Rightarrow \omega = \sqrt{\frac{L_1 + L_2}{\omega^2 L_1 L_2 C}}$$

2. **Previous Midterm** *(by Prof. Simon)*. An electric field in air E_p, polarized at 45 degrees in the x-y plane and moving in the -z direction can be written as

$$\vec{E}(z,t) = (\hat{x} + \hat{y})E_0 cos(\underbrace{kz}_{\phi(z)} + \omega t) \quad (1)$$

Here, $E_0 = E_p cos(\frac{\pi}{4}) = E_p sin(\frac{\pi}{4})$. The linearly polarized light is incident on a birefringent crystal, which has has two different indexes of refraction, n_x for the electric field aligned with the x-axis and n_y for the electric field along the y-axis.
(a) Find the crystal thickness z which will cause a $2\pi/4$ (or quarter wave) phase difference for the two polarizations of light with a free space wavelength of λ_0. **[10]**



$\phi(z) = kz$

$k_i = \left(\frac{2\pi}{\lambda_0}\right) \cdot n_i$

$\Rightarrow \Delta\phi(z) = z \cdot \Delta k = z\left(\frac{2\pi}{\lambda_0}\right)(n_x - n_y)$

$\Delta\phi(z_0) = \frac{2\pi}{4} = z_0 \frac{2\pi}{\lambda_0}(n_x - n_y)$

$\Rightarrow z_0 = \frac{\lambda_0}{4(n_x - n_y)}$

(b) Assume the crystal thickness is a quarter wave plate for a free-space λ_0, shifting the x- and y-components by 90 degrees. Write an expression for the electric field after the light exits the crystal. Assume that $n_x > n_y$ in the crystal. **[10]**

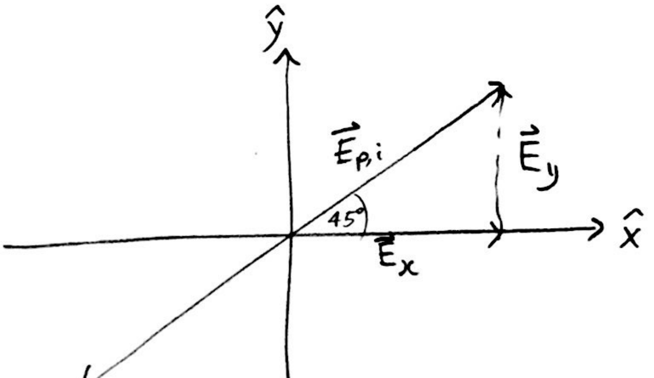
$n_x > n_y$, so λ_x will contract more in medium.

$\Rightarrow \vec{E}_x$ will lead $\vec{E}_y$ by $\pi/2$. $\Rightarrow \vec{E} = (\hat{x} \cdot e^{i\frac{\pi}{2}} + \hat{y}) E_0 e^{i(kz+\omega t)}$

$= (\hat{y} + i\hat{x}) E_0 e^{i(kz+\omega t)}$

$\Rightarrow$ circularly Polarized.

(c) A $40\mu m$ layer of cellophane tape acts as a half wave plate (π phase difference) for red light passing through it. Show graphically by vector addition that the initial 45 degree linear polarization of the entering wave is rotated by 90 degrees by a half wave plate. **[6]**



$\Rightarrow$