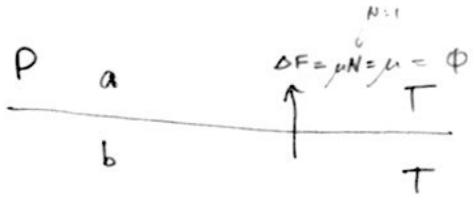
Q10

p.1




$\langle N_a \rangle = ?$

$$\underbrace{dF_a = dF_b}_{\text{evaporation}} \quad (\text{equilibrium})$$

$$\frac{\partial F_a}{\partial N_a} = \frac{\partial F_b}{\partial N_b} = \mu_a = \mu_b$$

$$\mu_b = \Phi$$

$$F_a = -kT\ln(Z_a)$$

$$F_b = -kT\ln(Z_b)$$

$$\frac{\partial F_a}{\partial N_a} = \frac{\partial F_b}{\partial N_b} = \Phi \Rightarrow \text{find } \langle N_a \rangle = \frac{\int N_a \, Z_{tot} \frac{\partial \rho}{h^3}}{\int Z_{tot} \frac{\partial \rho}{h^3}}$$

$$Z_a = \frac{1}{N_a!}\left( Z_{a_1} \right)^{N_a} \qquad Z_{a_1}$$

$$Z_b = \frac{1}{N_b!}\left( Z_{b_1} \right)^{N_b}$$

$$Z_{a_1} = \frac{gV4\pi}{h^3}\int_0^\infty p^2\,dp\; e^{-\beta p^2/2m} = \frac{V}{\lambda^3}$$

$$Z_{b_1} = \frac{gA2\pi}{h^2}\int_0^\infty p\,dp\; e^{-\beta(\varepsilon-\mu)} = \frac{A}{\lambda^2}e^{\beta\mu} = \frac{A}{\lambda^2}e^{\beta\Phi}$$

Q10

p.2

$$F_a = -kT\ln(Z_a) = -kT N_a \ln(Z_{a1}) + \underbrace{kT\ln N_a!)}_{kT(N_a\ln N_a - N_a)}$$

$$\frac{\partial F_a}{\partial N_a} = -kT\ln\left(\frac{V}{\lambda^3}\right) + kT\left(\ln N_a + 1 - 1\right)$$

$$= -kT\ln\left(\frac{V}{N_a\lambda^3}\right)$$

$$\frac{\partial F_b}{\partial N_b} = -kT\ln\left(\frac{A}{\lambda^2 N_b}\right) - \phi$$

$$\Rightarrow \quad \frac{\partial F_a}{\partial N_a} = \frac{\partial F_b}{\partial N_b} \Rightarrow e^{\phi\beta}\cdot\frac{A}{\lambda^2 N_b} = \frac{V}{N_a\lambda^3}$$

$$\Rightarrow N_a = \frac{VN_b}{A\lambda}e^{-\phi\beta} = \frac{VN_b}{A\lambda}e^{-\phi/kT}$$

$P_v$ $\quad \downarrow \frac{dN_a}{dT}$ $\quad T_0+\Delta T$

$P_v$ $\quad\quad\quad T_e = T_v$

$$\frac{\Delta N_a}{\Delta T} = \frac{\partial N_a}{\partial T}, \quad \lambda = \lambda_0 T^{-1/2}$$

$\uparrow T_{vapor}$

$T_{liquid} = const.$

$$N_a(T) = \frac{VN_b(T)}{A\lambda_0}\underbrace{T^{1/2}}_{f(T_v)} e^{-\frac{\Phi}{kT}}$$

$$\Rightarrow N_a(T+\Delta T) = \left(\frac{VN_b(T)}{A\lambda_0}\right) f(T+\Delta T)\; e^{-\frac{\Phi}{kT}}$$

$$f(T+\Delta T) \approx f(T) + \frac{\partial f}{\partial T}\Delta T + O(\Delta T^2) =$$

$$= f(T) + \frac{\Delta T}{2\sqrt{T}} + O(\Delta T^2)$$

$$\Rightarrow \boxed{\Delta N_a \approx \frac{\Delta T}{2}\cdot\frac{VN_b}{A\lambda\cdot T}e^{-\Phi/kT}}$$