Name:

Q1

10. *Electromagnetism*

A small sphere of polarizability α and radius a is placed at a great distance from a conducting sphere of radius b, which is maintained at a potential V relative to infinity. Find an approximate expression for the force on the dielectric sphere valid for $r \gg a$.

$$\vec{\nabla}.\vec{E}(r) = -\nabla^2\phi = \frac{q_{enc}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}.$$

$$\phi(r) = \frac{k_e Q}{r} = \frac{V}{r/b} \qquad \phi(r) = \frac{k_e Q}{b(r/b)}$$

$$Q = \frac{Vb}{k_e}$$

$$\Rightarrow \vec{E}(r) = -\vec{\nabla}\phi(r) = \frac{Vb}{r^2}$$

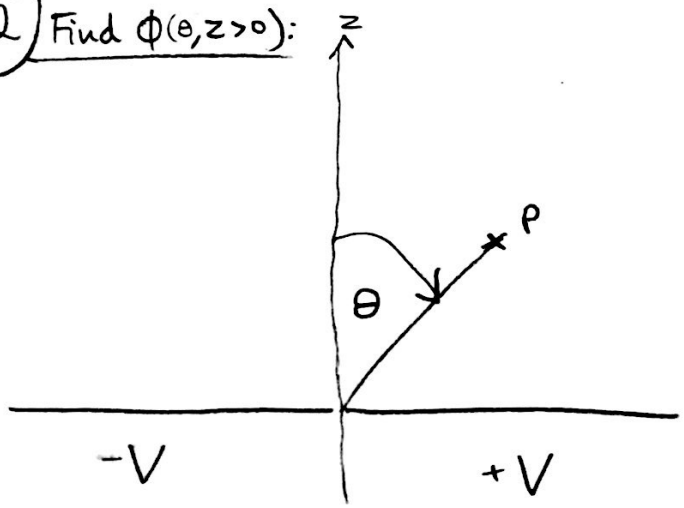
$$\vec{P} = \underbrace{\alpha}_{\varepsilon_0\chi_e}\vec{E} \quad ; \quad \vec{D} = \varepsilon_0\vec{E} + \varepsilon_0\chi_e\vec{E} = \varepsilon_0(1+\chi_e)\vec{E} = \varepsilon_0\varepsilon_r\vec{E} = \vec{D}$$

$$U = -\vec{p}.\vec{E} \quad ; \quad \vec{p} = \tfrac{4}{3}\pi a^3\vec{P} \Rightarrow U = -\tfrac{4}{3}\pi a^3\alpha|\vec{E}|^2 = \frac{-\frac{4}{3}\pi a^3\alpha b^2V^2}{r^4}$$

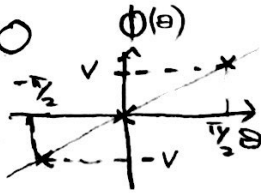
$$\vec{F} = -\frac{\partial U}{\partial r}\hat{r} = \frac{-16\pi a^3\alpha b^2V^2}{3r^5}\hat{r}$$

$$\boxed{F(r_0) = \frac{-16\pi a^3\alpha b^2V^2}{3r_0^5}\hat{r}}$$

Q2 Find $\phi(\theta, z>0)$:



$\phi(z>0, \theta=0) = 0$



$\Rightarrow \phi \propto \theta$, since

$$\Rightarrow \phi(\theta) = \frac{\theta}{\pi/2}V$$

$$\boxed{\phi(\theta) = \frac{2\theta}{\pi}V}$$