F'07 Q3

$$|\psi(t=0)\rangle = |\nu_e\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$$

$$|\psi(t)\rangle = e^{-iHt/\hbar}\,|\nu_e\rangle$$

$$P(|\nu_\mu\rangle, x=L) = ?$$, where $|\nu_\mu\rangle = -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle$

and $H|\nu_i\rangle = \sqrt{p^2c^2 + m_i^2c^4}\,|\nu_i\rangle$

$$P(|\nu_\mu\rangle, x=L) = \left|\langle \nu_\mu | \psi(t=\tfrac{L}{c})\rangle\right|^2 \quad ; \quad \sqrt{p^2c^2+m_i^2c^4} \approx pc + \frac{1}{2}\frac{m_i^2c^4}{pc}$$

$$|\psi(t=\tfrac{L}{c})\rangle = \cos\theta\, e^{-i\left(pc + \frac{1}{2}\frac{m_1^2c^4}{pc}\right)\frac{L}{c\hbar}}|\nu_1\rangle + \sin\theta\, e^{-i\left(pc+\frac{1}{2}\frac{m_2^2c^4}{pc}\right)\frac{L}{c\hbar}}|\nu_2\rangle$$

$$= e^{\frac{-ipL}{\hbar}}\, e^{-\frac{i}{2}\frac{m_1^2c^4L}{pc^2\hbar}}\left[\cos\theta\,|\nu_1\rangle + \sin\theta\, e^{-\underbrace{\frac{i}{2}\frac{\Delta m^2c^2}{p}\frac{L}{\hbar}}_{\alpha}}|\nu_2\rangle\right]$$

$$\Rightarrow \left|\langle\nu_\mu|\psi(t=\tfrac{L}{c})\rangle\right|^2 = \left|-\sin\theta\cos\theta + \cos\theta\sin\theta\, e^{-i\alpha}\right|^2 =$$

$$= \left[\frac{1}{2}\sin(2\theta)\right]^2 \left|e^{-i\alpha} - 1\right|^2 =$$

$$= \frac{1}{4}\sin^2(2\theta)\big(\underbrace{\cos^2\alpha - 2\cos\alpha + 1 + \sin^2\alpha}_{2-2\cos\alpha \,=\, 2(1-\cos^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}) \,=\, 4\sin^2(\frac{\alpha}{2})}\big) \Rightarrow$$

$$= \sin^2(2\theta)\,\sin^2\left(\frac{\Delta m^2c^2L}{4p\hbar}\right)$$

$$\Rightarrow \boxed{P(|\nu_\mu\rangle, x=L) = \sin^2(2\theta)\,\sin^2\left(\frac{\Delta m^2c^2L}{4p\hbar}\right)}$$