

F'07 Q3

$$|\psi(t=0)\rangle = |v_e\rangle = \cos\theta |v_1\rangle + \sin\theta |v_2\rangle$$

$$|\psi(t)\rangle = e^{-iHt/\hbar} |v_e\rangle$$

$$P(|v_\mu\rangle, x=L) = ? \quad , \quad \text{where } |v_\mu\rangle = -\sin\theta |v_1\rangle + \cos\theta |v_2\rangle$$

$$\text{and } H|v_i\rangle = \sqrt{p^2 c^2 + m_i^2 c^4} |v_i\rangle$$

$$P(|v_\mu\rangle, x=L) = |\langle v_\mu | \psi(t=\frac{L}{c}) \rangle|^2 \quad , \quad \sqrt{p^2 c^2 + m_i^2 c^4} \approx pc + \frac{1}{2} \frac{m_i^2 c^4}{pc}$$

$$\begin{aligned} |\psi(t=\frac{L}{c})\rangle &= \cos\theta e^{-i\left(pc + \frac{1}{2} \frac{m_1^2 c^4}{pc}\right) \frac{L}{c\hbar}} |v_1\rangle + \sin\theta e^{-i\left(pc + \frac{1}{2} \frac{m_2^2 c^4}{pc}\right) \frac{L}{c\hbar}} |v_2\rangle \\ &= e^{-\frac{i p L}{\hbar}} e^{-\frac{i}{2} \frac{m_1^2 c^4 L}{p c^2 \hbar}} \left[\cos\theta |v_1\rangle + \sin\theta e^{-\frac{i \Delta m^2 c^2 L}{2 p \hbar}} |v_2\rangle \right] \end{aligned}$$

$$\begin{aligned} \Rightarrow |\langle v_\mu | \psi(t=\frac{L}{c}) \rangle|^2 &= \left| -\sin\theta \cos\theta + \cos\theta \sin\theta e^{-i\alpha} \right|^2 = \\ &= \left[\frac{1}{2} \sin(2\theta) \right]^2 \left| e^{-i\alpha} - 1 \right|^2 = \\ &= \frac{1}{4} \sin^2(2\theta) \underbrace{(\cos^2\alpha - 2\cos\alpha + 1 + \sin^2\alpha)}_{2 - 2\cos\alpha = 2(1 - \cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2})} \Rightarrow \\ &= \frac{1}{4} \sin^2(2\theta) \underbrace{2(1 - \cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2})}_{= 4 \sin^2(\frac{\alpha}{2})} \\ &= \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 c^2 L}{4 p \hbar}\right) \end{aligned}$$

$$\Rightarrow P(|v_\mu\rangle, x=L) = \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 c^2 L}{4 p \hbar}\right)$$