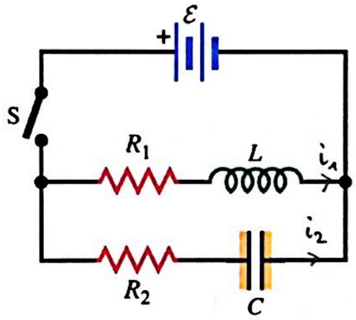
Name: 1C Discussion- Week 4

1. **YF 30.79**. Switch S is closed at time $t = 0$, causing a curent i_1 through the inductive branch and a current i_2 through the capacitive branch. The initial charge on the capacitor is zero, and the charge at time t is q_2.
(a) Write the circuit equations using Kirchoff's laws.
(b) What is the initial current through the inductive branch and the capacitative branch? What are the currents through the branches at $t = \infty$?
(c) Write expressions for i_1, i_2, and q_2 as functions of time. Draw plots for each.
(d) After a long time, the switch S is opened. Describe qualitatively what happens to the current through the circuit. What is the frequency of damped oscillations?



a) $\mathcal{E} = R_1 i_1 + L \frac{di_1}{dt} = R_2 i_2 + \frac{q_2}{C}$

b) $i_1(t=0) = 0$ $\qquad i_1(t=\infty) = \frac{\mathcal{E}}{R_1}$

$i_2(t=0) = \frac{\mathcal{E}}{R_2}$ $\qquad i_2(t=\infty) = 0$

c) $i_1(t) = ?$, $i_2(t) = ?$, $q_2(t) = \int_0^t i_2(t)\,dt$

$i_1(t)$: $\mathcal{E} = R_1 i_1 + L \frac{di_1}{dt}$

$$\left(i_1 - \mathcal{E}/R_1\right) = -\frac{L}{R_1}\frac{di_1}{dt}$$

$$\int_{i_1(0)=0}^{i_1(t)} \frac{1}{i_1 - \mathcal{E}/R_1}\, di_1 = -\int_0^t \frac{R_1}{L}\, dt$$

$$\ln\left(\frac{i_1(t) - \mathcal{E}/R_1}{0 - \mathcal{E}/R_1}\right) = -\frac{R_1}{L} t$$

$$\boxed{i_1(t) = \mathcal{E}/R_1 \left(1 - e^{-\frac{R_1}{L}t}\right)}$$

$i_2(t)$: $\mathcal{E} = R_2 i_2 + \frac{q_2}{C}$ $\Big) \frac{d}{dt}$

$$0 = R_2 \frac{di_2}{dt} + \frac{i_2}{C}$$

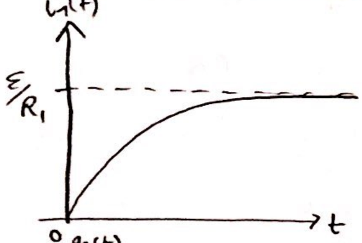
$$\Rightarrow \int_{i_2(0)}^{i_2(t)} \frac{1}{i_2}\, di_2 = -\int_0^t \frac{1}{R_2 C}\, dt$$

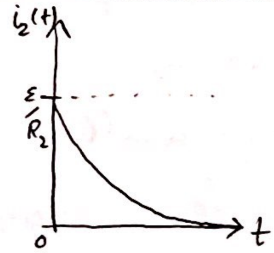
$$\ln\left(\frac{i_2(t)}{\mathcal{E}/R_2}\right) = -\frac{t}{R_2 C}$$

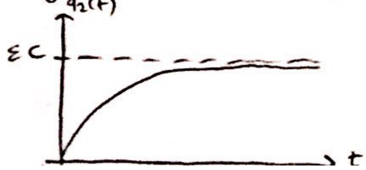
$$\boxed{i_2(t) = \mathcal{E}/R_2\, e^{-\frac{t}{R_2 C}}}$$

$q_2(t)$: $\Rightarrow q_2(t) = \int_0^t \mathcal{E}/R_2\, e^{-\frac{t}{R_2 C}}\, dt$

$$\Rightarrow \boxed{q_2(t) = \mathcal{E}C\left(1 - e^{-\frac{t}{R_2 C}}\right)}$$







2. **RLC series.** Consider an RLC series circuit (where the elements are arranged in order $R \to L \to C$) with an alternating voltage source $v_s(t) = V_s cos(\omega t)$. Suppose that V_{out} denotes the voltage accross the capacitor.
a) Determine the ratio V_{out}/V_S.
b) Determine the leading order behavior of this ratio for $\omega \to 0$ and $\omega \to \infty$.
c) Can you think of a way that these results can be used in real life?

a) $V_{out} = I X_c = \frac{1}{\omega C}$

$$V_s = I X_{RLC} = I\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\frac{V_{out}}{V_s} = \frac{1}{\omega C\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

b) $$\frac{V_{out}}{V_s} = T(\omega) = \frac{1}{C\sqrt{\omega^2 R^2 + \left(\omega^2 L - \frac{1}{C}\right)^2}}$$

$$\left.\frac{V_{out}}{V_s}\right|_{\omega=0} = T(0) = 1 \Rightarrow V_{out}(\omega=0) = V_s$$

$$\left.\frac{V_{out}}{V_s}\right|_{\omega=\infty} = T(\infty) = 0 \Rightarrow V_{out}(\omega=\infty) = 0$$

c) This is an example of a "low-pass filter", since output voltage is about the same as the source voltage for low frequencies, but is very small for high frequencies. In other words, it lets low frequencies "pass" and blocks high frequencies.
A "high-pass filter" takes Vout as the voltage accross R+L.