Problem 4.1. Consider a system of $N \gg 1$ non-interacting particles in which the energy of each particle can assume two and only two distinct values: 0 and E ($E > 0$). Denote by n_0 and n_1 the occupation numbers of the energy levels 0 and E, respectively. The fixed total energy of the system is U.

a) Find the entropy of the system.

b) Find the temperature as a function of U. For what range of values of n_0 is $T < 0$?

c) In which direction does heat flow when a system of negative temperature is brought into thermal contact with a system of positive temperature? Why?

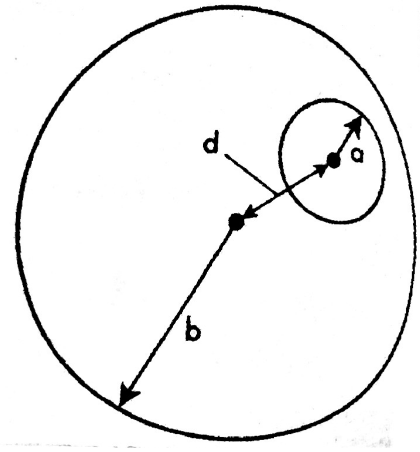
22. In a hot plasma, all the atoms may be regarded as completely ionized. Although the ions have long-range forces due to Coulomb interactions, macroscopically the plasma is electrically neutral. This suggests that the Coulomb interactions are screened, and so become short-range. Estimate this range, making suitable approximations.

12. The average energy of a system in thermal equilibrium is $\langle E \rangle$. Prove that the mean square deviation of the energy from $\langle E \rangle$, $\langle (E - \langle E \rangle)^2 \rangle$ is given by

$$\langle (E - \langle E \rangle)^2 \rangle = kT^2 C_v,$$

where C_v is the heat capacity of the entire system at constant volume. Use this result to show that the energy of a macroscopic system may ordinarily be considered constant when the system is in thermal equilibrium.

7. An eccentric hole of radius a is bored parallel to the axis of a right circular cylinder of radius b ($b > a$). The two axes are at a distance d apart. A current of I amperes flows in the cylinder. What is the magnetic field at the center of the hole?



23. Inside a superconductor, instead of Ohm's Law ($\mathbf{J} = \sigma\mathbf{E}$), we assume London's equations to be valid for the current density $\mathbf{J}$:

$$c \,\mathbf{curl}\, (\lambda\mathbf{J}) = -\mathbf{B}, \qquad \frac{\partial}{\partial t}(\lambda\mathbf{J}) = \mathbf{E}$$

(in Gaussian units), and regard λ as a constant. Otherwise, Maxwell's equations (with $\epsilon = 1$, $\mu = 1$) and the corresponding boundary conditions are unchanged.

Consider an infinite superconducting slab of thickness $2d$ ($-d < z < d$), outside of which there is a given constant magnetic field parallel to the surface:

$$H_x = H_z = 0,\ H_y = H_0 \qquad (\textit{same}\ \text{value for}\ z > d\ \text{and}\ z < -d),$$

with $\mathbf{E} = \mathbf{D} = 0$ everywhere. If surface currents and charges are absent, compute $\mathbf{H}$ and $\mathbf{J}$ inside the slab.

21. The ionosphere can be considered as an ionized medium containing N essentially free electrons per unit volume. Show that if a linearly polarized wave propagates in the ionosphere in a direction parallel to that of the small uniform magnetic field **H** produced by the earth, its plane of polarization will be rotated through an angle proportional to the distance traveled by the wave. Calculate the constant of proportionality.

42. Two thin, parallel, infinitely long, nonconducting rods, a distance a apart, with identical constant charge density λ per unit length in their rest frame, move with a velocity v, not necessarily small compared to the speed of light. Calculate the force per unit length between them in a frame of reference that is at rest, and in a frame of reference moving with the rods, and compare the results.