QI. (a) If neutrons from a cosmic-ray interaction one light-year from the earth were to reach here with a probability of $1/e$ or greater, what must their minimum energy be? (b) If they then decay, what is the maximum angle to the flight path at which their decay electrons could be produced? (c) What is the maximum angle for the decay neutrinos? (d) At the angle calculated in (c), what is the maximum energy of the neutrino?

Problem 1.9. A bead of mass m slides without friction on a circular loop of radius a. The loop lies in a vertical plane and rotates about a vertical diameter with constant angular velocity ω (Figure 1.5).

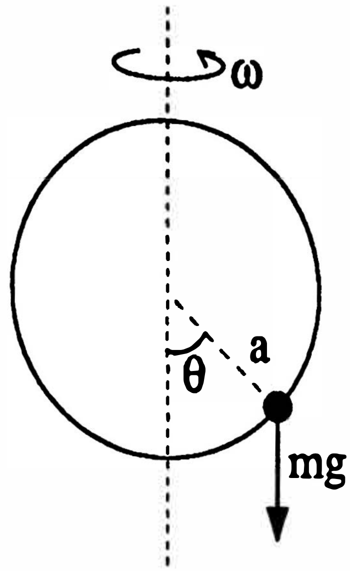

Figure 1.5.

a) For angular velocity ω greater than some critical angular velocity ω_c, the bead can undergo small oscillations about some stable equilibrium point θ_0. Find ω_c and $\theta_0(\omega)$.

b) Obtain the equations of motion for the small oscillations about θ_0 as a function of ω and find the period of the oscillations.

Problem 3.10. An isolated hydrogen atom has a hyperfine interaction between the proton and electron spins ($\mathbf{S}_1$ and $\mathbf{S}_2$, respectively) of the form $J\mathbf{S}_1 \cdot \mathbf{S}_2$. The two spins have magnetic moments $\alpha\mathbf{S}_1$ and $\beta\mathbf{S}_2$, and the system is in a uniform static magnetic field $\mathbf{B}$. Consider only the orbital ground state.

a) Find the exact energy eigenvalues of this system and sketch the hyperfine splitting spectrum as a function of magnetic field.

b) Calculate the eigenstates associated with each level.

Problem 3.6. a) A spin 1/2 electron is in a uniform magnetic field $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$. At time $t = 0$ the spin is pointing in the x-direction, i.e., $\langle S_x(t = 0)\rangle = \hbar/2$. Calculate the expectation value $\langle \mathbf{S}(t)\rangle$ at time t.

b) An additional magnetic field $\mathbf{B}_1 = \frac{1}{2}B_1[\cos(\omega t)\hat{\mathbf{x}} + \sin(\omega t)\hat{\mathbf{y}}]$ is now applied. If an electron in the combined field $\mathbf{B}_0 + \mathbf{B}_1$ has spin pointing along $+\hat{\mathbf{z}}$ at time $t = 0$, what is the probability that it will have flipped to $-\hat{\mathbf{z}}$ at time t?

Name:

Questions for the Comprehensive Exam Fall 2013

3. *Quantum Mechanics: Scattering*

Consider the scattering of a spinless particle of mass m from a diatomic molecule. The incoming particle travels along the z-axis. Assume that the molecule is much heavier than the scattering particle and that there is no recoil. The two atoms in the molecule are aligned along the y-axis and localized at y = +b and y = −b. The potential the particle feels in the presence of the molecule can be modeled by the following potential:

$$V(\vec{x}) = \alpha\big(\delta(y-b)\delta(x)\delta(z) + \delta(y+b)\delta(x)\delta(z)\big)$$

(a) Calculate the scattering amplitude in the first Born approximation.
(b) Calculate the differential cross section from (a) (Express the result in terms of the scattering angles).
(c) Calculate the total cross section. Do the integrals exactly. You might find the following integrals helpful:

$$\int_0^{2\pi} d\alpha \left|\cos(x\sin\alpha)\right|^2 = \pi\big(1 + J_0(2x)\big), \qquad \int_0^{\pi} d\alpha (\sin\alpha) J_0(x\sin\alpha) = \frac{a\sin x}{x}$$

6. *Quantum Mechanics*

A particle in a spherically symmetrical potential is known to be in an eigenstate of $\mathbf{L}^2$ and L_z with eigenvalues $\hbar^2 l(l+1)$ and $m\hbar$, respectively, denoted by $|lm\rangle$. $\mathbf{L}$ is the angular momentum operator, whose components obey the usual commutation algebra. Prove that the expectation values involving L_x and L_y obey

$$\langle L_x \rangle = \langle L_y \rangle = 0, \quad \langle L_x^2 \rangle = \langle L_y^2 \rangle = \frac{l(l+1)-m^2}{2}\hbar^2$$

in the eigenstate $|lm\rangle$.

2. Three matrices M_x, M_y, M_z, each with 256 rows and columns, are known to obey the commutation rules $[M_x, M_y] = iM_z$ (with cyclic permutations of x, y, z). The eigenvalues of one matrix, say M_x, are ± 2, each once; $\pm\frac{3}{2}$ each 8 times; ± 1, each 28 times; $\pm\frac{1}{2}$, each 56 times; 0,70 times. State the 256 eigenvalues of the matrix $M^2 = M_x^2 + M_y^2 + M_z^2$.

Part b) **16.** Consider the one-dimensional Schrödinger equation with

$$V(x) = \begin{cases} \frac{m}{2}\omega^2 x^2 & \text{for } x > 0, \\ +\infty & \text{for } x < 0. \end{cases}$$

Find the energy eigenvalues.