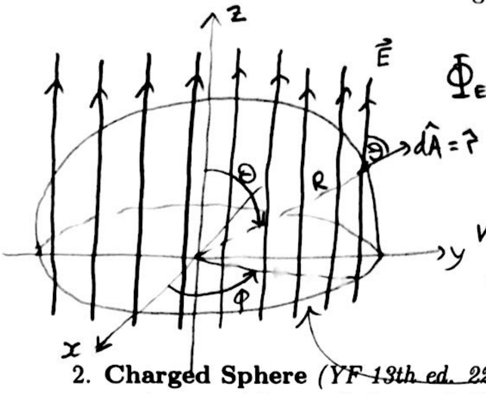
Name: 1B Discussion - Week

1. **Out of left field** *(YF 13th ed. 22.5)*. A hemispherical surface with radius r in a region of uniform electric field $\vec{\mathbf{E}}$ has its axis aligned parallel to the direction of the field. Calculate the flux through the surface.

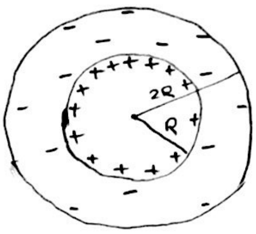


$$\Phi_E = \oint \underbrace{\vec{E}\cdot d\vec{A}}_{|E||dA|\cos\theta} = \underbrace{\int_0^{\pi/2} \sin\theta\cos\theta \, d\theta}_{1/2} \underbrace{\int_0^{2\pi} d\varphi}_{2\pi} \cdot ER^2 = E\cdot\pi R^2$$

$$\underbrace{dA}_{} = R^2 \sin\theta \, d\theta \, d\varphi$$

nb/ Flux through surface of hemisphere equals flux through base area $\Phi_E = EA = E\pi R^2$, since $Q_{encl.} = 0$

2. **Charged Sphere** *(YF 13th ed. 22.50)*. A solid conducting sphere with radius R carries a positive total charge Q. The sphere is surrounded by an insulating shell with inner radius R and outer radius $2R$. The insulating shell has a uniform charge density ρ. (a) Find the value of ρ so that the net charge of the entire system is zero. (b) If ρ has the value found in part (a), find the electric field (magnitude and direction) in each of the regions $0 < r < R$, $R < r < 2R$, and $r > 2R$. Show your results in a graph of the radial component of $\vec{\mathbf{E}}$ as a function of r. (c) As a general rule, the electric field is discontinuous only at locations where there is a thin sheet of charge. Explain how your results in part (b) agree with this rule.



a) $\rho = ? =$ uniform $\Rightarrow$ constant

$$\int_R^{2R} \rho \, 4\pi r^2 dr = -Q = \frac{4}{3}\pi(8-1)R^3\rho$$

$$\Rightarrow \rho = -\frac{3Q}{28\pi R^3}$$

b)

$$E(r<R,\ r>2R) = 0$$

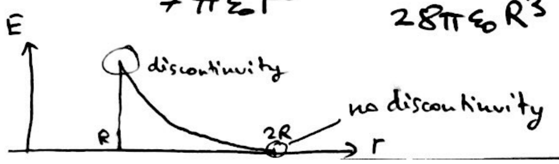
$E(R<r<2R) = ?$; Gauss' law: $\int_R^r E\cdot dA = \frac{Q_{encl}}{\varepsilon_0} = E\,4\pi r^2$

$$Q_{encl.} = Q + \int_R^r \rho\, 4\pi r^2 dr = Q + \frac{4\pi}{3}\rho(r^3 - R^3)$$

$$\Rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2} + \frac{\rho}{3\varepsilon_0 r^2}(r^3 - R^3)$$

$$\Rightarrow E = \frac{2Q}{7\pi\varepsilon_0 r^2} - \frac{Qr}{28\pi\varepsilon_0 R^3}$$

c)

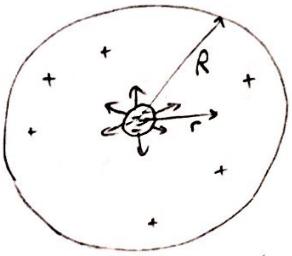


chargesheet on surface of conductor leads to discontinuity in ρ, E. ■

3. **The historical Thomson model** *(YF 13th ed. 22.54)*. In the early years of the 20th century, a leading model of the structure of the atom was that of the English physicist J. J. Thomson (the discoverer of the electron). In Thomson's model, an atom consisted of a sphere of positively charged material in which were embedded negatively charged electrons, like chocolate chips in a ball of cookie dough. Consider such an atom consisting of one electron with mass m and charge $-e$, which may be regarded as a point charge, and a uniformly charged sphere of charge $+e$ and radius R. (a) Explain why the equilibrium position of the electron is at the center of the nucleus. (b) In Thomson's model, it was assumed that the positive material provided little or no resistance to the motion of the electron. If the electron is displaced from equilibrium by a distance less than R, show that the resulting motion of the electron will be simple harmonic, and calculate the frequency of oscillation. (c) By Thomson's time, it was known that excited atoms emit light waves of only certain frequencies. In his model, the frequency of emitted light is the same as the oscillation frequency of the electron or electrons in the atom. What would the radius of a Thomson-model atom have to be for it to produce red light of frequency $4.57 \times 10^{14} Hz$? Compare your answer to the radii of real atoms, which are of the order of $10^{-10}m$. (d) If the electron were displaced from equilibrium by a distance greater than R would the electron oscillate? Would its motion be simple harmonic? Explain your reasoning. (Historical note: In 1910, the atomic nucleus was discovered, proving the Thomson model to be incorrect. An atom's positive charge is not spread over its volume as Thomson supposed, but is concentrated in the tiny nucleus of radius $10^{-14}m$ to $10^{-15}m$.)

nb/ JJ. found that $e^- =$ particle.
His son, G.P. Thomson found that $e^- =$ wave, also.
Both got the Nobel prize for their discoveries.

a)



The equilibrium is where there is no net force on the electron.
At the center of the charged sphere, all forces cancel.

b) $$E_r = \frac{er}{4\pi\varepsilon_0 R^3}$$ from $$4\pi r^2 E = \frac{\rho \, \frac{4}{3}\pi r^3}{\varepsilon_0}$$, where $\rho = \frac{e}{\frac{4}{3}\pi R^3}$ = uniform.

$$F = -eE = -\frac{1}{4\pi\varepsilon_0}\frac{e^2 r}{R^3} = -kr = -m\omega^2 r \Rightarrow \omega = \sqrt{\frac{e^2}{4\pi\varepsilon_0 m R^3}}$$

c) $$f = 4.57\times10^{14}\ Hz = \frac{\omega}{2\pi} \Rightarrow R = \left(\frac{1}{4\pi\varepsilon_0}\frac{e^2}{4\pi^2\varepsilon_0 m\omega^2}\right)^{1/3} = 3.13\times10^{-10}\ m$$

↳ agrees (atom radi

d) It would still oscillate, because the force is directed towards the center of "atom".
The force at $r > R$ scales as $F \propto \frac{1}{r^2}$ ⇒ will not be S.H.M..