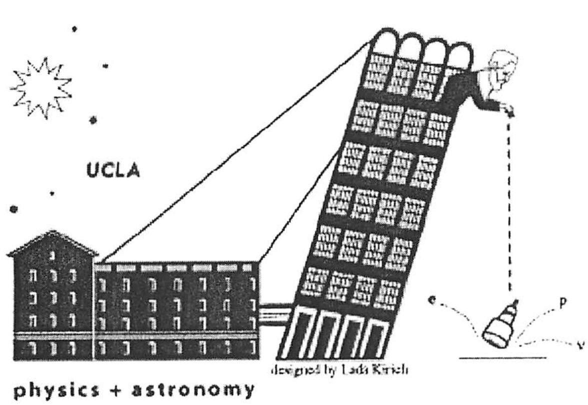
Spring 2008
Physics Comprehensive Exam

March 26, 2008 (Part 1) 9:00 - 1:00pm

Part 1: Quantum Mechanics and Statistical Mechanics
7 Total Problems/20 Points Each/Total 140 Points

- Closed book exam.
- Calculators not allowed.
- Use paper provided for each problem. <u>Use one side only.</u>
- Print your name and page number on every page.
- Return the problem page as the first page of your answers.
- When submitting, please clip all pages together in problem # order.
- If a part of any problem seems ambiguous to you, state clearly your interpretations and answer the question accordingly.



Quantum Mechanics Problem #1

Name:

Consider a charged particle of charge e and mass m in a constant crossed electric and magnetic field

$$\vec{E} = (0, 0, E), \quad \vec{B} = (0, B, 0), \quad \vec{r} = (x, y, z)$$

a) Write the Schrödinger equation (in a convenient gauge).

b) Separate variables and reduce it to a one-dimensional problem.

c) Calculate the expectation value of the velocity in the x direction in any energy eigenstate (sometimes called the drift velocity).

Name:

Quantum Mechanics Problem #2

Helium is a two-electron atom described by the Hamiltonian

$$H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} - \frac{2e^2}{r_1} - \frac{2e^2}{r_2} + H'$$

where H' is the electron-electron interaction. Electron spin will be relevant, but we are neglecting spin dependent forces. Imagine a world where, instead of the realistic Coulomb interaction between electrons, we have

$$H' = \frac{\lambda \mathbf{r_1} \cdot \mathbf{r_2}}{r_1 \cdot r_2}$$

a) Ignoring the perturbation H', what is the energy of the first excited level (the level next above the ground level)? What is the degeneracy?

Now switch on H', treating it to first order in perturbation theory. The degeneracy is partially lifted by the perturbation, the unperturbed level under discussion splitting into several levels, each with its own remaining degeneracy.

b) How many such distinct sublevels are there? What is the degeneracy of each?

c) One of the sublevels has shifted in energy by an aount $\Delta > 0$. In terms of this Λ, what are the shifts of the other sublevels?

Name:

Quantum Mechanics Problem #3

The beam of atoms of mass m and energy ϵ is passed through a small hole of diameter d in an opaque plate normal to the beam. The atoms are then detected by a second plate a distance of L away. Under the most idealistic circumstances, what is the lower bound for the diameter D of the spot the beam forms on the detector plate? L and ϵ are fixed, but the hole diameter d may be varied. Take the beam cross section to be always much wider than the hole. Notice that as $d \rightarrow 0$ the uncertainty in momentum will lead to $D \rightarrow \infty$, while as $d \rightarrow \infty$ clearly D will also become very large. You are asked to find the optimal hole size d for which D is the smallest.

Quantum Mechanics
Problem #4

Assume a charged particle of charge e and mass m in a uniform external magnetic field $\vec{B} = B\hat{e_z}$. The system will exhibit quantized energy states (Landau levels).

a) Write down the Hamiltonian of this system. Use $\vec{A} = Bx\vec{e}_y$.

b) Consider the unitary operator $\hat{U} = \exp(i\frac{p_x p_y}{\hbar m\omega_0})$, with $\omega_0 = \frac{eB}{mc}$. How does the Hamiltonian transform under this unitary transformation, $H' = \hat{U}H\hat{U}^\dagger$? What's the physical meaning of ω_0?

Hint: You may find the following formulae useful:

$$e^{\lambda B}Ae^{-\lambda B} = A + \sum_{n=1}^{\infty}\frac{\lambda^n}{n!}[B,A]_n, \quad [B,A]_n = [B,[B,A]_{n-1}] \tag{5}$$

c) Find the energy eigenvalues of H'.

Name:

Quantum Mechanics Problem #5

Consider a one-dimensional scattering problem: An electron at energy ϵ is incident on a square potential barrier of height W and length d. At what energies ϵ will electrons pass through the well without any reflection?

Name:

Statistical Mechanics Problem #6

Estimate the pressure P at the nose of a supersonic jet, if the Mach number (i.e. the ratio of the aircraft speed to the speed of sound) is $M = 5$. The air pressure at the altitude of the flight is P_0. Treat the air compression near the aircraft to be adiabatic satisfying

$$\frac{P}{\rho^\gamma} = \text{const.}$$

where ρ is a variable gas density and γ is a constant. Assume the gas flow very close to the jet nose to have a vanishing speed relative to it.

Hint: Approximate the process of gas flow in front of the aircraft as a compression near a piston closing at a steady speed in a very long cylinder. Far from the piston the gas is stationary but in front of it there is a steady-state disturbance in density and pressure, which propagates along with the piston. Try to understand the hydrodynamics and thermodynamics of this disturbance. Recall that the speed of sound is in general given by $s = \sqrt{dP/d\rho}$ (with the derivative taken for the adiabatic compression). Use linearlized equations for the hydrodynamics of gas flow.

Name:

Statistical Mechanics Problem #7

An isolated volume can be separated in two parts, V_1, and V_2, by inserting a wall. In each part of the volume there are N particles of an ideal mono-atomic gas. The temperatures T_1, T_2 in both parts are adjusted such that the pressures $P_1 = P_2 = P_0$ are identical in each part.

a) Now the wall is removed. Compute the temperature T and the pressure P in the equilibrium state.

b) Compute the change of entropy as a function of T_1, T_2 and N. What happens if $T_1 = T_2$?

Statistical Mechanics Problem #8

Name:

Calculate the drag force on a disc of radius R moving with constant speed v (perpendicular to the plane of the disc) in a dilute gas of density n that is in thermal equilibrium at temperature T. Assume that the gas molecules collide elastically with the disc, that the speed of the disc is slow compared to the average molecular speed, and that the disc is large compared to a molecule, but small compared to the mean free path of the electrons.

Name:

Statistical Mechanics Problem #9

Find the temperature at which the equilibrium distribution function for the velocity of hydrogen molecules (H_2) will be the same as that of the nitrogen molecules (N_2) at room temperature. Recall that the most abundant isotope of hydrogen contains a single proton while that of nitrogen 7 protons and 7 neutrons.

Name:

Electromagnetics Problem #10

a) Starting from the (nonrelativistic) Larmor formula for the total power radiated by an accelerated charge:

$$P = \frac{2}{3}\frac{q^2}{c^3}\left|\frac{dv}{dt}\right|^2$$

write a relativistic generalization for this power which

- is a Lorentz scalar;
- reduces to the Larmor formula for $\beta \to 0$ (show that your invariant does that).

b) Now take a relativistic ($\gamma \gg 1$) charged particle in uniform circular motion and calculate the total power radiated in terms of the radius R of the orbit and the energy E of the particle.

c) For the case of a 10 GeV electron in a 100m radius circular accelrator, calculate the energy radiated per revolution in eV (within a factor 2 is good enough).

Name:

Electromagnetics Problem #11

A particle of mass m and charge e is suspended on a string of length L above an infinite plane conductor. The distance of closest approach is a. There is no gravity.

a) Compute the frequency of this pendulum for small oscillations.

b) Compute the power radiated per unit solid angle, $dP/d\Omega$, as a function of angles, for small oscillations of linear amplitude X_0.

c) Compute the total power radiated (take the case $\lambda \gg a$), in terms of e, X_0, a, m, L.

Name:

Electromagnetics Problem #12

A "coaxial cable" consists of two concentric, perfectly conducting, infinite cylinders of radii a and b $(a < b)$.

a) Using the Maxwell equations and appropriate boundary conditions, show that a *transverse, non-homogeneous*, e.m. *plane* wave can propagate down the cable in the space between the cylinders (which we take to be filled with vacuum). Non-homogeneous means that the amplitude is not homogeneous in space.

b) Characterize the wave, i.e. find the **E** and **B** fields (e.g. in terms of the max value of $|\mathbf{E}| = E_m$) and the relation between ω and k (the dispersion relation). Are all frequencies allowed?

Name:

Electromagnetics Problem #13

An electromagnetic wave is incident at the Brewster angle (the angle at which the reflected wave is polarized) on an interface where the wave impedance changes from Z_1 to Z_2. Show that the power transmission coefficient $T = 4Z_1Z_2\cos\theta_1\cos\theta_2/[Z_1\cos\theta_1 + Z_2\cos\theta_2]^2$ is unity.

Electromagnetics Problem #14

Name:

An electromagnetic wave in free space has electric field components given by $E_x = 2E_Q \cos(kz - \omega t)$ and $E_y = E_0 \sin(kz - \omega t)$. Calculate the components of $\vec{H}$ and plot H_x vs H_y. Find the instantaneous Poynting vector and its maximum, minimum and average value.