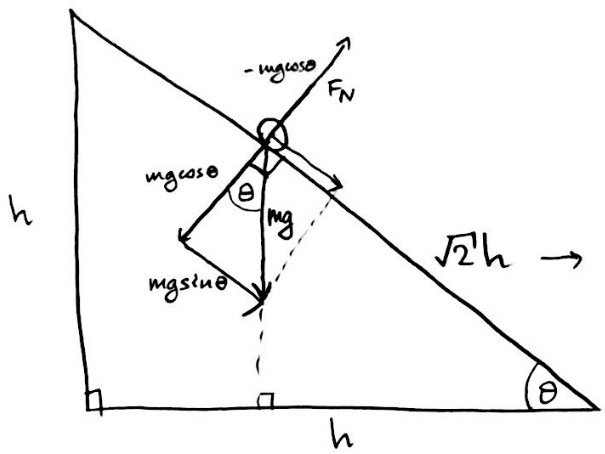
1. **Galileo's Experiment** *(A Briefer History of Time, Hawking, 2005).* According to popular belief, Galileo dropped two objects of different weights from the tower of Pisa to show that they accelerate uniformly and equally. While this is probably untrue, Galileo is documented to have rolled spheres of different weights down an incline to disprove the Aristotelian view that objects of different masses accelerate differently. Assume Galileo used an incline of equal height and length, $h = 10m$.
(a) Draw a free body diagram of the sphere rolling from the top of the incline.
(b) Find the time it takes for a sphere of any mass to roll down. Does this time depend on the mass of the sphere?
[Useful facts: $\arctan(1) = \pi/4$; $\sin(\pi/4) = 1/\sqrt{2}$; grav. acceleration, $g \approx 10m/s^2$]

a)



$\sqrt{2}h \rightarrow$ Pythagoras: $h^2 + h^2 = \Delta x^2$

$\Rightarrow \Delta x = \sqrt{2}h$

b)

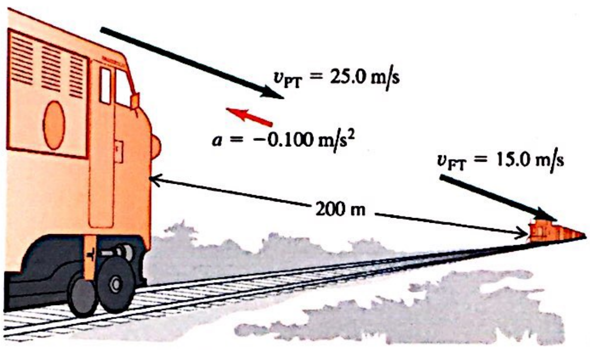
$F_{net} = ma = mg\sin\theta \quad \Rightarrow a = g\sin\theta$

$\theta = \tan^{-1}\left(\frac{h}{h}\right) = \frac{\pi}{4} \quad \Rightarrow a = \frac{g}{\sqrt{2}}$

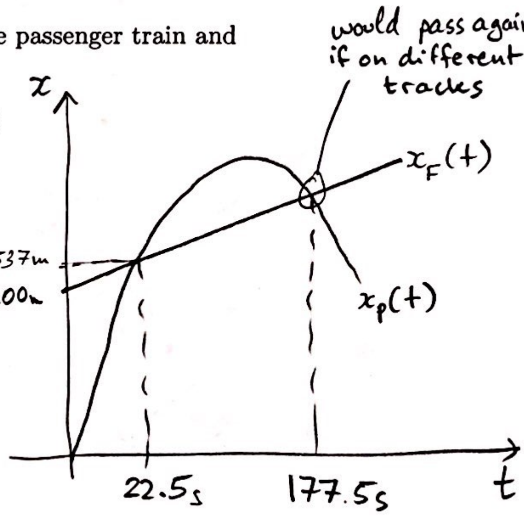
$x = x_0 + vt + \frac{1}{2}at^2 \quad \Rightarrow \Delta x = \cancel{vt} + \frac{1}{2}at^2$

$\Rightarrow t = \sqrt{\frac{2\Delta x}{a}} = \sqrt{\frac{2\sqrt{2}\cdot 10\cdot\sqrt{2}}{10}} = \sqrt{4} = 2$

2. **Train Collision?** *(YF 13th edition 2.70).* The engineer of a passenger train traveling at 25.0 m/s sights a freight train 200m ahead on the same track. The freight train is traveling at 15.0 m/s in the same direction as the passenger train. The engineer of the passenger train immediately applies the brakes causing a constant acceleration of $0.100 m/s^2$ in a direction opposite to the train's velocity. Take $x = 0$ at the location of the front of the passenger train when the engineer applies the brakes.
(a) Will the cows nearby witness a collision?
(b) If so, where will it take place?
(c) On a single graph, sketch the positions of the front of the passenger train and the back of the freight train.



c)



a) Collision if $x_{PT}(t) = x_{FT}(t)$.

$$x_P(t) = \cancel{x_{0,P}}^{\,0} + \underbrace{v_P t}_{25 m/s \cdot t} + \underbrace{\tfrac{1}{2} a_P t^2}_{-0.05 t^2 \cdot m/s^2} \quad , \quad x_F = \underbrace{x_{0,F}}_{200 m} + \underbrace{v_F t}_{15 m/s\, t} + \cancel{\tfrac{1}{2} a_F t^2}^{\,0}$$

⇒ Set $x_P = x_F$ and see if answer makes sense:

$$v_P t + \tfrac{1}{2} a_P t^2 = x_{0,F} + v_F t$$

$$\Rightarrow \quad \tfrac{1}{2} a_P t^2 + (v_P - v_F) t - x_{0,F} = 0$$

Solve like any quadratic formula:

$$t_{1,2} = \frac{-10 \pm \sqrt{100 - 40}}{-0.1} \quad \Rightarrow \quad t_1 = 22.5s > 0 \Rightarrow \text{collision!}$$

b) $x_P = 25 m/s \cdot 22.5s - 0.05 (22.5s)^2 m/s^2 = 537\ m$ from origin.