F'13Q9

$\omega^2 = \gamma k^4$

$\omega(k) = \gamma_0 k^2$ (dispersion relation)

We are dealing with bosons (# of waves unlimited): $n(\varepsilon) = \frac{1}{e^{+\hbar\omega\beta} - 1}$

$C = \frac{dE}{dT} \Rightarrow$ find $E(T)$.

$$E = \frac{R}{2\pi\hbar} \int_0^{2\pi}\int_{-\infty}^{\infty} dp\, dq \; \frac{\hbar\omega}{e^{+\hbar\omega\beta} - 1} =$$

$q = \phi$

$p = \hbar k = \hbar\sqrt{\frac{\omega}{\gamma_0}}$

$dp = \frac{1}{2}\frac{\hbar\, d\omega}{\sqrt{\omega\gamma_0}}$

$$E = \frac{R}{2\pi\hbar}\, 2\pi \int_{-\infty}^{\infty} \frac{1}{2}\frac{\hbar^2}{\sqrt{\gamma_0}} \frac{\omega^{+1/2}\, d\omega}{e^{+\hbar\omega\beta} - 1}$$

$\hbar\omega\beta = y \qquad \omega^{+1/2} = y^{+1/2}\frac{1}{\sqrt{\hbar\beta}}$

$d\omega = \frac{1}{\hbar\beta} dy$

$$E = \frac{R\hbar}{2\sqrt{\gamma_0}}\left(\frac{1}{\hbar\beta}\right)^{3/2} \int_{-\infty}^{\infty} \frac{y^{+1/2}\, dy}{e^{+y} - 1} = \frac{R k^{3/2}}{2\sqrt{\hbar\gamma_0}}\, T^{+3/2}\, Z(+\tfrac{1}{2}) \Rightarrow$$

$$C = \frac{dE}{dT} = \frac{R k^{3/2}}{2\sqrt{\hbar\gamma_0}}\left(\frac{3}{2}T^{1/2}\right) Z(+\tfrac{1}{2}) \Rightarrow \boxed{C = \frac{3k^{3/2}}{4}\frac{R\, Z(\tfrac{1}{2})}{\sqrt{\hbar}\,(\gamma)^{1/4}}\, T^{+\frac{1}{2}}}$$