Physics 1A: Physics for Scientists and Engineers: Mechanics	Winter 2015

Discussion 7: Week 9

Table 7.1: Comparison of Angular Kinetics and Linear Kinetics

Linear Kinetics	Angular Kinetics
$x = x_0 + v_0 t + at^2/2$	$\theta = \theta_0 + \omega_0 t + \alpha t^2/2$
$v = \mathrm{d}x/\mathrm{dt} = v_0 + at$	$\omega = \mathrm{d}\theta/\mathrm{dt} = \omega_0 + \alpha t$
$a = \mathrm{d}v/\mathrm{dt} = \mathrm{d}^2x/\mathrm{dt}^2$	$\alpha = \mathrm{d}\omega/\mathrm{dt} = \mathrm{d}^2\theta/\mathrm{dt}^2$
$v_f^2 - v_i^2 = 2a(x_f - x_i)$	$\omega_f^2 - \omega_i^2 = 2\alpha(\theta_f - \theta_i)$
m	$I = \sum_i m_i r_i^2$
K.E.$= \frac{1}{2}mv^2$	K.E.$= \frac{1}{2}I\omega^2$

Exercise 1 A computer disk drive is turned on starting from rest and has constant angular acceleration. If it took time t_2 for the driver to make its second complete revolution, (a) how long did it take to make the first complete revolution, and (b) what is its angular acceleration?

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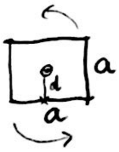
b) $\theta_0 = \omega_0 = 0$

$\theta(t_2) = 4\pi = \frac{\alpha t_2^2}{2} \Rightarrow \alpha = \frac{8\pi}{t_2^2}$

a) $\theta(t_1) = 2\pi = \frac{4\pi t_1^2}{t_2^2} \Rightarrow t_1^2 = \frac{t_2^2}{2} \Rightarrow t_1 = \frac{t_2}{\sqrt{2}}$

Exercise 2 A thin, uniform rod is bent into a square of side length a. If the total mass is M, find the moment of inertia about an axis through the center and perpendicular to the plane of the square. (Moment of inertia of a slender rod with axis through center is $I = \frac{1}{12}ML^2$)



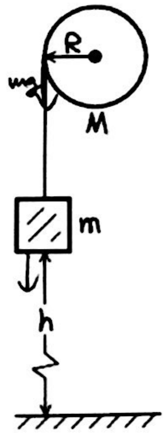
$$I_1 = Md^2 + I_{rod} = \frac{M}{4}\cdot\left(\frac{a}{2}\right)^2 + \frac{1}{12}\frac{Ma^2}{4}$$

$$I = 4I_1 = M\frac{a^2}{4} + M\frac{a^2}{12} = \frac{Ma^2}{3}$$

Exercise 3 In the system shown in the figure on the right, a mass m is released from rest and falls, causing the uniform cylinder of mass M, diameter R to turn about a frictionless axle through its center. How far will the mass have to descend to give the cylinder a kinetic energy of E? (moment of inertia for a cylinder rotating about its center axis is given by $I = \frac{1}{2}MR^2$)

$K = \frac{1}{2}I\omega^2$

$I =$



$$mgh = \frac{1}{2}m\underset{\omega r}{v^2} + \underbrace{\frac{1}{2}I\omega^2}_{E}$$

$$= \frac{1}{2}m\omega^2R^2 + E = \frac{1}{2}m\frac{2KR^2}{I} + K$$

$$\omega = \sqrt{\frac{2K}{I}} =$$

$$\Rightarrow h = K\left(\frac{1}{mg} + \frac{R^2}{Ig}\right)$$

$$= K\left(\frac{1}{mg} + \frac{2}{Mg}\right)$$

$$= K\left(\frac{M+2m}{Mmg}\right)$$

or $\omega = v/R$, $\frac{1}{2}I\omega^2 = \frac{1}{4}Mv^2$

$$h = \frac{\frac{1}{2}mv^2 + \frac{1}{4}Mv^2}{mg}$$