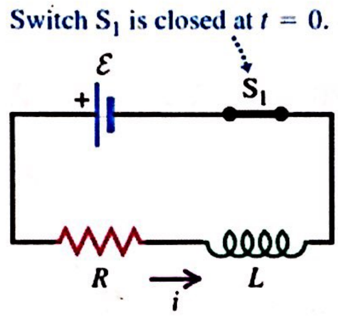
1. **LR Circuit** *(YF 12th ed. Fig. 30.12)*. Calculate the current in the circuit as a function of time, $I(t)$. [Hint: Apply Kirchoff's rule to find the circuit equation; determine initial and final currents; integrate with respect to time.]



a) $\mathcal{E} = RI + L\dfrac{dI}{dt}$

b) $I(t=0) = 0$

$I(t=\infty) = \dfrac{\mathcal{E}}{R}$

c) $\dfrac{L}{R}\dfrac{dI}{dt} = \dfrac{\mathcal{E}}{R} - I$

$$\frac{\frac{dI}{dt}}{\frac{\mathcal{E}}{R} - I} = \frac{R}{L}$$

$$\int_{I=0}^{I(t)} \frac{1}{\frac{\mathcal{E}}{R} - I}\, dI = \int_0^t \frac{R}{L}\, dt$$

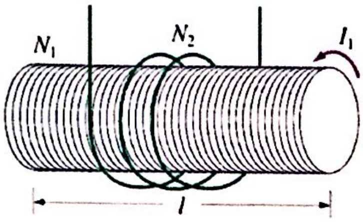
$$-\left[\ln\left(\mathcal{E}/R - I(t)\right) - \ln\left(\mathcal{E}/R\right)\right] = \frac{Rt}{L}$$

$$\Rightarrow \frac{\mathcal{E}/R - I(t)}{\mathcal{E}/R} = e^{-\frac{Rt}{L}}$$

$$\Rightarrow I(t) = \mathcal{E}/R\left(1 - e^{-\frac{Rt}{L}}\right) \Rightarrow \begin{array}{l} I(0) = 0 \\ I(\infty) = \mathcal{E}/R \end{array}$$

✓

2. **Previous Midterm** *(by Prof. Simon)*. The figure below shows two coils. Coil 1 is a long straight solenoid with N_1 windings and cross sectional area A. Coil 2 has N_2 windings wrapped along a short length of the solenoid. What is the mutual inductance M due to the current I_1 in the solenoid?



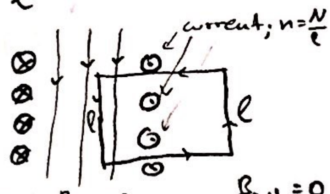
$\Phi_2 \propto I_1 \Rightarrow N_2 \Phi_2 \propto I_1$ or

$N_2 \Phi_2 = M_{21} I_1$

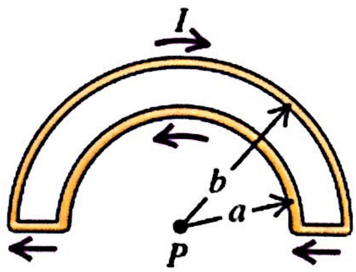
$$\Rightarrow M_{21} = \frac{N_2 \Phi_2}{I_1} = \frac{N_2 B_1 A}{I_1} = \frac{N_2 N_1 B_1 A \mu_0}{l}$$

$$B_1 = \mu_0 n_1 I_1 = \frac{\mu_0 N_1 I_1}{l}$$ for solenoid

$B \cdot \cancel{l} = \mu_0 n \cancel{l} I$



3. **Biot-Savart** *(YF 13th ed. 28.74)*. The figure below shows two semicircles with radii a and b. Calculate the net magnetic field (magnitude and direction) that the current in the wire produces at P.



$r\,d\theta\,\hat{s}$

$$\vec{dB} = \frac{\mu_0 I}{4\pi} \frac{\vec{dl} \times \hat{r}}{r^2}$$

$$B = \frac{\mu_0 I}{4\pi} \left[\int_0^{\pi} \frac{1}{a} d\theta - \int_0^{\pi} \frac{1}{b} d\theta \right]$$

$$= \frac{\mu_0 I}{4\pi} \left(\frac{\pi}{a} - \frac{\pi}{b} \right)$$

$$= \frac{\mu_0 I}{4} \left(\frac{1}{a} - \frac{1}{b} \right)$$