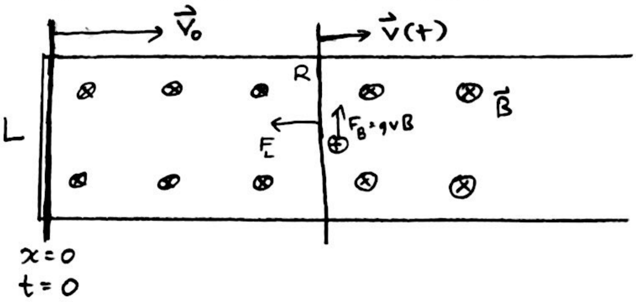
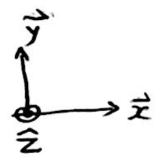
Name: 1C Discussion- Week 2

1. **Rail problem.** A pair of rails run along the x-direction and are separated by a distance L. They are connected at some point by a stationary wire. To the right of the stationary wire a slide-wire completes the circuit. The slide-wire can move along the rails. A uniform magnetic field $\mathbf{B} = -B\hat{z}$ points into the page. The slide-wire has mass m, resistance R and initial position $x_0 = 0$.
(a) Find the force F on the slide-wire when it is given a velocity $\mathbf{v} = v_0\hat{x}$.
(b) Solve for the motion of the wire as a function of time. What is the behavior of the velocity and position of the wire at large t?
(c) The circuit in the wire dissipates power as $P = \epsilon I$. Where does this energy come from? Calculate the total energy dissipated (as $t \to \infty$). Does this agree with your theory of where the energy came from?





a) $\vec{F}_L = ?$

(1) Current direction: $\vec{F}_B = q\,\vec{v}\times\vec{B} = (qvB)\,\hat{y}$

(2) Lorentz Force: $\vec{F}_L = q\frac{d\vec{y}}{dt}\times\vec{B} = \vec{y}\underbrace{\frac{dq}{dt}}_{I}\times\vec{B} = I\,\vec{y}\times\vec{B} = I\vec{L}\times\vec{B} = -ILB\,\hat{x}$

$\Rightarrow F_L = ILB \;,\; \hat{F}_L = -\hat{x}$

↳ what is I?

$$I = \frac{U}{R} = \frac{|\varepsilon|}{R}$$

↳ what is $|\varepsilon|$?

$$\varepsilon = -\frac{d\Phi_B}{dt} \;;\; \Phi_B = \vec{B}.\vec{A} = BA = BLx(t)$$

$$= -BL\frac{dx}{dt} = -BL\,v(t)$$

$|\varepsilon| = BLv$

$$\Rightarrow \boxed{\vec{F}_L = -\frac{B^2L^2v}{R}\,\hat{x}}$$

Note, that for a superconducting slide-wire ($R \to 0$) any non-zero $v(t)$ will cause $F_L \to \infty$, resisting motion. The slide-wire is frozen into magnetic space, keeping Φ = const.

b) $\vec{F}_{net} = -\frac{B^2L^2 v}{R}\,\hat{x} = m\frac{dv}{dt}\,\hat{x}$

$$\Rightarrow \quad m\frac{dv}{dt} = -\frac{B^2L^2v}{R}$$

$$\Rightarrow \quad \frac{1}{v}\frac{dv}{dt} = -\frac{B^2L^2}{mR}$$

$$\Rightarrow \quad \int_{v_0}^{v(t)} \frac{1}{v}\,dv = -\int_0^t \frac{B^2L^2}{mR}\,dt = -\underbrace{\frac{B^2L^2}{mR}}_{k}\,t = -kt$$

$$\Rightarrow \quad \ln(v(t)) - \ln(v_0) = \ln\left(\frac{v(t)}{v_0}\right) = -kt$$

$$\boxed{v(t) = v_0 e^{-kt}}$$

$$x(t) = \int_0^t v(t)\,dt = -\frac{v_0}{k}\left[e^{-kt} - e^{-k\cdot 0}\right] = \underbrace{\frac{v_0}{k}}_{x_\infty = \text{final position}}\left[1 - e^{-kt}\right]$$

$$\boxed{x(t) = x_\infty\left[1 - e^{-kt}\right]}$$

c) $P = \varepsilon I = \frac{\varepsilon^2}{R} = \frac{B^2L^2v^2}{R}$;

$$\Delta E = \int_0^\infty P(t)\,dt = \int_0^\infty \frac{B^2L^2v^2(t)}{R}\,dt = \frac{B^2L^2v_0^2}{R}\int_0^\infty e^{-\frac{2B^2L^2}{mR}t}\,dt =$$

$$= -\frac{1}{2}mv_0^2\left[e^{-\infty} - e^0\right] =$$

$$= \frac{1}{2}mv_0^2 = E_{initial}$$

✓