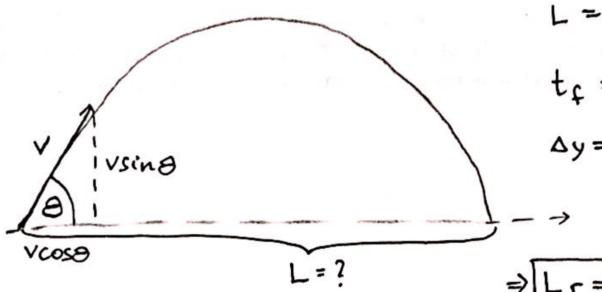
1. **Uniform accelerated motion.** Conventional artillery launches projectiles of mass m at an angle θ with muzzle velocity v. Given that angle, at what range L can it hit a target? A railgun can deliver more than 6 times the conventional velocity, $v_r = 6v$. How does the railgun range L_r compare to L?



$L = v_x t_{flight} = v\cos\theta\, t_f$

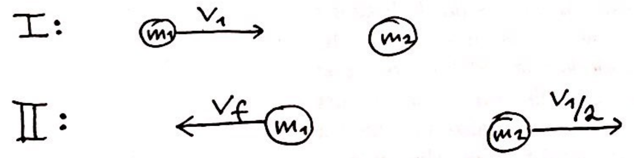
$t_f = ?$

$\Delta y = 0 = v_y t_f - \frac{1}{2} g t_f^2 \Rightarrow t_f = \frac{2v_y}{g}$

$L = v\cos\theta \frac{2v\sin\theta}{g} = \frac{2\sin\theta\cos\theta}{g} v^2 \propto v^2$

$\Rightarrow \boxed{L_r = 36L}$

2. **Energy and momentum.** Consider the following elastic collision in 1D. Puck 1, of mass m_1, moving with initial velocity v_1, hits puck 2 (of mass m_2), which is intially at rest. (a) Find m_2 in terms of m_1 such that the final velocity of puck 2 is $v_1/2$. (b) What is the final velocity v_f of puck 1?



$p = \text{cons.} = m_1 v_1 = m_2 v_1/2 + m_1 v_f$

$E = \text{cons.} = \frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_2 \frac{v_1^2}{4} + \frac{1}{2} m_1 v_f^2$

$\rightarrow v_f = v_1\left(1 - \frac{m_2}{2m_1}\right) \rightarrow$ plug into E = cons.

$\rightarrow m_1 v_1^2 = m_2 \frac{v_1^2}{4} + m_1 v_1^2\left(1 - \frac{m_2}{m_1} + \frac{m_2^2}{4m_1^2}\right)$

$\cancel{m_1^2 v_1^2} = m_1 \cancel{m_2} \frac{2 v_1^2}{4} + \cancel{m_1^2 v_1^2} - m_1 \cancel{v_1^2} \cancel{m_2} + \frac{m_2^{\cancel{2}} \cancel{v_1^2}}{4}$

$\Rightarrow \boxed{m_2 = 3m_1}$

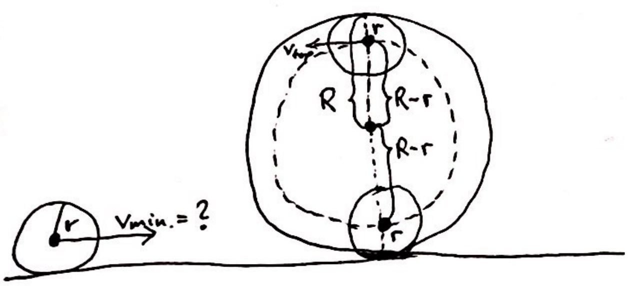
$\Rightarrow \boxed{v_f = -\frac{v_1}{2}}$

3. **Rotation.** A rolling solid sphere enters a circular hoop of radius R. How fast (v) does it need to roll to make it all the way around? The answer should take into account R_{loop} and r_{sphere}. The moment of inertia of a solid sphere is $I = (2/5)mr^2$.

For point particle, $\frac{mv^2}{R} = mg\cos\theta + N$

At top, without normal force (=weightless), $v_{top}^2 = gR$.

For solid sphere, $v_{top}^2 = g(R-r)$, using center of mass as effective concentration of sphere mass.



$E = \text{cons.} = E_i = \frac{1}{2} m v_{min.}^2 + \frac{1}{2} I\omega_{min}^2 = \underbrace{\frac{1}{2} m v_{top}^2 + \frac{1}{2} I \omega_{top}^2}_{KE_{top}} + mg\overbrace{2(R-r)}^{\Delta h} = E_f$

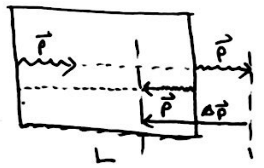
$\omega = \frac{v}{r}, \; I = \frac{2}{5} mr^2$

$\Rightarrow E_i = \frac{1}{2} m v_{min}^2 + \frac{1}{5} mr^2 \frac{v_{min}^2}{r^2} = \frac{7}{10} m v_{min}^2 \;\; \frac{7}{10} m v_{top}^2 = KE_{top} = \frac{7}{10} mg(R-r).$

$\Rightarrow \frac{7}{10} m v_{min}^2 = \frac{7}{10} mg(R-r) + mg2(R-r) = \frac{27}{10} mg(R-r) \Rightarrow \boxed{v_{min}^2 = \frac{27}{7} g(R-r)}$

4. **Impulse.** In a laser, light is trapped between two parallel mirrors (i.e. light bounces back and forth between the mirrors). Light consists of particles (photons) each carrying a momentum $p = h\nu/c$ (ν is the frequency of the light, c the speed of light, h a fundamental constant of nature which is called Planck's constant, [units of angular momentum]). Photons travel at speed c. Suppose the distance between the mirrors is L and there is only one photon in the laser. (a) Calculate the (average) force exerted by the light on one mirror. (b) If we increase the distance between the mirrors to 2L, what is the work done by this "radiation force"?

a)

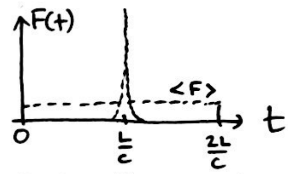


$\vec{p}$ is a vector, $|\vec{p}|$ is conserved but $\Delta\vec{p} = 2|\vec{p}| = \frac{2h\nu}{c}$

$F = ma = m\frac{dv}{dt} = \frac{dp}{dt} \Rightarrow \Delta p = \int_0^{t_{coll.}} F(t)\,dt$,

$\hookrightarrow \Delta p = \int_0^{\frac{2L}{c}} \langle F\rangle\,dt$

$\hookrightarrow \boxed{\langle F\rangle = \frac{h\nu}{L}}$



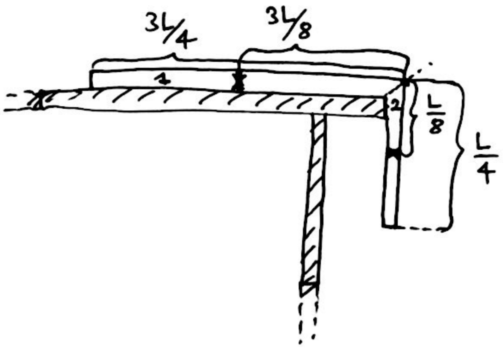
one collision every $t_{coll.} = \frac{2L}{c}$.

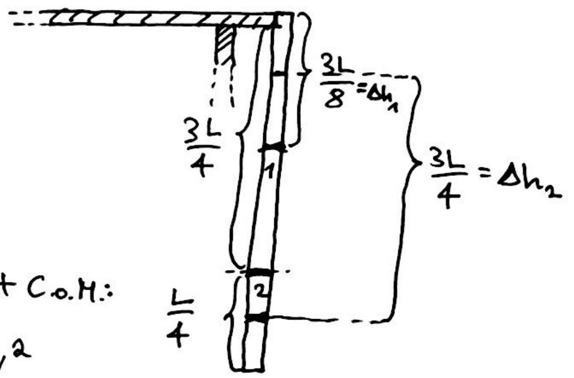
$F(t) \approx \langle F\rangle$, constant.

b) Careful: $\langle F\rangle$ is a function of L.

$\boxed{W = \int_L^{2L} \langle F\rangle\,dL = h\nu\ln(2)}$

5. **Challenge Question.** A uniform rope of mass m and length L is partly hanging over the edge of a frictionless table. Initially, 1/4 of the rope is hanging over the edge (the rest is stretched out on the table). The rope is released from rest and slides down. (a) Calculate the velocity of the rope when the trailing end is just over the edge of the table. (b) Same question, but now there is also a coefficient of dynamic friction μ_d between the rope and (the horizontal part of) the table.





a) Divide rope into two objects, centered at C.o.M.:

$E = \text{cons.} = m_1 g\Delta h_1 + m_2 g\Delta h_2 = \frac{1}{2}Mv^2$

$m_1 = \frac{3}{4}M$, $\Delta h_1 = \frac{3L}{8}$

$m_2 = \frac{1}{4}M$, $\Delta h_2 = \frac{3L}{4}$

$\Big\}\; v^2 = 2\left(\frac{9}{32}L + \frac{6}{32}L\right)g \Rightarrow \boxed{v = \sqrt{\frac{15}{16}gL}}$

b) $\frac{9}{32}MLg + \frac{3}{16}MLg = \frac{1}{2}Mv^2 + \frac{3}{4}Mg\cdot\left(\frac{3L}{8}\right)\mu \Rightarrow \boxed{v = \sqrt{\frac{15}{16}gL - \frac{9}{16}g\mu L}}$