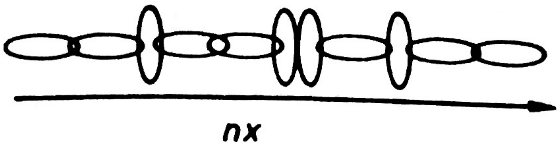


Fig. 2.4.

Solution:

(a) When the length of the chain is nx, there are $m = nx/a$ segments parallel to the chain; so the microscopic state number is

$$\Omega = C_m^n = \frac{n!}{m!(n-m)!} .$$

We have

$$\begin{aligned} S &= k \ln \Omega \\ &= k \ln \frac{n!}{\left(\frac{x}{a}n\right)! \left(n - \frac{x}{a}n\right)!} . \end{aligned}$$

(b) Under the action of stress F, the energy difference between the vertical and parallel states of a segment is Fa. The mean length of a segment is

$$l = \frac{ae^{Fa/kT}}{1 + e^{Fa/kT}} ,$$

so that

$$nx = nl = \frac{nae^{Fa/kT}}{1 + e^{Fa/kT}} .$$

(c) At high temperatures,

$$L = nx = na\left(\frac{1}{2} + \frac{1}{2}\frac{Fa}{kT}\right) ,$$

which is Hooke's Law.

QUESTION 4 [14 points]

(a) 3 pts

For any system of fermions at chemical potential μ and temperature T, show that the probability for finding an occupied state of energy $\varepsilon + \mu$ is the same as that for finding an unoccupied state of energy $\mu - \varepsilon$.

Consider now a system of non-interacting Dirac fermions of spin 1/2 and mass m. One-particle states at momentum $\mathbf{p}$ come in pairs of positive and negative energy,

$$\varepsilon_\pm(\mathbf{p}) = \pm\sqrt{m^2c^4 + \mathbf{p}^2c^2}$$

for each value of the spin quantum number. At $T = 0$, all negative energy Dirac states are filled (the so-called Dirac-sea), and all positive energy states are empty, so that $\mu(T = 0) = 0$.

(b) 3 pts Using the result of (a) compute the chemical potential at arbitrary temperature T.

(c) 3 pts Compute (an integral representation for) the mean excitation energy $E(T) - E(0)$

(d) 2 pts Evaluate the integral in part (c) for $m = 0$ and evaluate the specific heat C_V;

(e) 3 pts Describe the qualitative change in the specific heat at low temperature when $m \neq 0$.

Solution to Questions 4

(a) The probabilities for finding occupied states at energy $\mu + \varepsilon$ and energy $\mu - \varepsilon$ are respectively given by,

$$n(\mu + \varepsilon) = \frac{1}{e^{\beta\varepsilon} + 1} \qquad\qquad n(\mu - \varepsilon) = \frac{1}{e^{-\beta\varepsilon} + 1}$$

which are clearly related by $n(\mu - \varepsilon) = 1 - n(\mu + \varepsilon)$, and the right side is precisely the probability for finding the state unoccupied at energy $\mu + \varepsilon$. QED.

(b) Total particle number is unchanged at finite temperature, and using the above particle-hole symmetry, the chemical potential remains zero at any temperature, $\mu(T) = 0$.

(c) Formally, the internal energy is given by summing over the contributions from positive and negative energy states, with their associated occupation numbers,

$$E(T) = 2V \int \frac{d^3\mathbf{p}}{(2\pi\hbar)^3}\Big(\varepsilon_+(\mathbf{p})\, n(\mu + \varepsilon_+(\mathbf{p})) + \varepsilon_-(\mathbf{p}) n(\mu + \varepsilon_-(\mathbf{p}))\Big)$$

where the prefactor of 2 arises because of the electron spin degeneracy. We now use $\varepsilon_-(\mathbf{p}) = -\varepsilon_+(\mathbf{p})$ as well as the result of (a), namely $n(\mu + \varepsilon_-(\mathbf{p})) = n(\mu - \varepsilon_+(\mathbf{p})) = 1 - n(\mu + \varepsilon_+(\mathbf{p}))$. Hence, we have

$$E(T) = 2V \int \frac{d^3\mathbf{p}}{(2\pi\hbar)^3}\Big(2\varepsilon_+(\mathbf{p})\, n(\mu + \varepsilon_+(\mathbf{p})) + \varepsilon_-(\mathbf{p})\Big)$$

Omitting the T-independent contribution of the term in $\varepsilon_-(\mathbf{p})$ in the integrand amounts to subtracting the energy of the negative energy Dirac sea. Also, setting now $\mu = 0$ by the result of (b), we find,

$$E(T) - E(0) = 4V \int \frac{d^3\mathbf{p}}{(2\pi\hbar)^3} \frac{\varepsilon_+(\mathbf{p})}{e^{\beta\varepsilon_+(\mathbf{p})} + 1}$$

The multiplicity is explained as follows. One factor of 2 arises from spin 1/2, while the other arises from the contributions of both positive and negative energy states.

(d) For $\varepsilon_+(\mathbf{p}) = c|\mathbf{p}|$, we have,

$$E(T) - E(0) = 4V \int \frac{d^3\mathbf{p}}{(2\pi\hbar)^3} \frac{c|\mathbf{p}|}{e^{\beta c|\mathbf{p}|} + 1} = \frac{2V}{\pi^2\hbar^3 c^3} (kT)^4 \int_0^\infty dx \, \frac{x^3}{e^x + 1}$$

The value of the last integral is $6\zeta(4) = \pi^4/15$, but it is not essential that it be evaluated.

(e) When $m \neq 0$, the energy spectrum develops a gap of size $2mc^2$, so that there must be a suppression factor $\exp\{-mc^2/kT\}$ at low temperatures.

Solution 4.2. a) In order to calculate the specific heat of a classical system, it is necessary only to know the number of degrees of freedom of the system. The specific heat then follows from an application of the equipartition theorem.

For the case of a heteronuclear diatomic molecule, there are some subtleties to do with quantum mechanics that we cannot ignore. Classically the angular momentum vector is free to point in any direction with respect to the orientation of the molecule, and since the rotational energy is $\frac{1}{2}(I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2)$, one might naively expect there to be three degrees of freedom. However, it is a consequence of quantum mechanics that a system *cannot* rotate about an axis of continuous symmetry, which in the case of a diatomic molecule means that there can be no component of angular momentum along the axis joining the two atoms (see Burcham). This microscopic constraint has a macroscopically observable effect, namely that the number of degrees of freedom of rotation is reduced from three to two. (Often the same result is "derived" classically, by arguing that the moment of inertia about the symmetry axis is zero.)

The classical average internal energy is given by

$$\langle E \rangle = (\#\text{ degrees of freedom}) \times \frac{1}{2}kT = kT, \tag{13.8}$$

where k is Boltzmann's constant. Therefore the specific heat is given by

$$C = \frac{d\langle E \rangle}{dT} = k. \tag{13.9}$$

b) The partition function $\mathcal{Z}$ is defined to be the sum of the Boltzmann factor $e^{-E/kT}$ over all distinct quantum states of the system. For a system with energy levels $E_j = (\hbar^2/2I)j(j+1)$, each having degeneracy $(2j+1)$, we find

$$\mathcal{Z} = \sum_{j=0}^{\infty}(2j+1)e^{-\beta E_j}, \tag{13.10}$$

where $\beta \equiv 1/kT$. The average energy $\langle E \rangle$ is defined as

$$\langle E \rangle = \frac{\sum\limits_{j=0}^{\infty} E_j(2j+1)e^{-\beta E_j}}{\sum\limits_{j=0}^{\infty}(2j+1)e^{-\beta E_j}} = -\frac{\partial}{\partial\beta}\ln\mathcal{Z}. \tag{13.11}$$

c) As $T \to 0$ $(\beta \to \infty)$, the occupation numbers of the higher excited states will be heavily suppressed by the Boltzmann factor, and for sufficiently low temperature we can replace the infinite sums in equations (13.10) and (13.11) by finite sums over just the ground state and the first excited state. We get

$$\mathcal{Z} \approx 1 + 3e^{-\beta E_1}, \tag{13.12}$$

so that

$$\ln \mathcal{Z} \approx 3e^{-\beta E_1}, \tag{13.13}$$

where we have assumed that $3e^{-\beta E_1} \ll 1$ and used the small-x expansion $\ln(1+x) \approx x$. Therefore, at low temperatures,

$$\langle E \rangle \approx -\frac{\partial}{\partial \beta} 3e^{-\beta E_1} = 3E_1 e^{-E_1/kT}. \tag{13.14}$$

We can differentiate this to find the low-temperature specific heat,

$$C = \frac{d\langle E \rangle}{dT} = \frac{3E_1^2}{kT^2} e^{-E_1/kT}. \tag{13.15}$$

We note that for small T, the exponential suppression dominates over the $1/T^2$ prefactor, and $C \to 0$ as $T \to 0$.

Our derivation is valid provided that the occupation number of the second level is far smaller than that of the first ($5e^{-\beta E_2} \ll 3e^{-\beta E_1}$), and that the occupation number of the first level is much smaller than unity ($3e^{-\beta E_1} \ll 1$). In fact both of these yield the same condition,

$$kT \ll \frac{\hbar^2}{I}. \tag{13.16}$$

d) In the limit $T \to \infty$ ($\beta \to 0$), many states will become heavily populated and contribute to the sums in the partition function (13.10) and the average energy (13.11). Under these circumstances it becomes legitimate to approximate the discrete sum by an integral, with vanishingly small error for large enough T. Therefore we can write

$$\mathcal{Z} \approx \int_0^\infty dj\,(2j+1) \exp\left(-\frac{\beta \hbar^2}{2I} j(j+1)\right). \tag{13.17}$$

We integrate to obtain

$$\mathcal{Z} \approx \frac{-2I}{\beta\hbar^2}\left[\exp\left(-\frac{\beta\hbar^2}{2I}j(j+1)\right)\right]_0^\infty = \frac{2I}{\beta\hbar^2}, \tag{13.18}$$

so that

$$\ln\mathcal{Z} \approx -\ln\beta + \text{constant.} \tag{13.19}$$

Thus we find for the average internal energy

$$\langle E\rangle = -\frac{\partial}{\partial\beta}\ln\mathcal{Z} \approx \frac{1}{\beta} = kT, \tag{13.20}$$

which we recognize as the classical result. Therefore the specific heat at high temperatures is simply

$$C = \frac{d\langle E\rangle}{dT} \approx k. \tag{13.21}$$

This result is valid provided the Boltzmann factor is large up to values of j much greater than unity, that is $\beta E_1 \ll 1$, or $kT \gg \hbar^2/I$. There is, of course, another limit to the validity of our expression, namely that the temperature must not be so high that the molecule dissociates.

Chapter 11

Electricity & Magnetism—Solutions

Solution 2.1. This problem combines two of the simplest geometries which textbooks use to demonstrate the use of image charges to solve boundary value problems, so we should not be surprised that image charge methods work in this case as well. Let the line through the charge q and the center of the bulge be the $\hat{\mathbf{z}}$-axis, with the origin such that the charge is at $z = p$ (Figure 11.1).

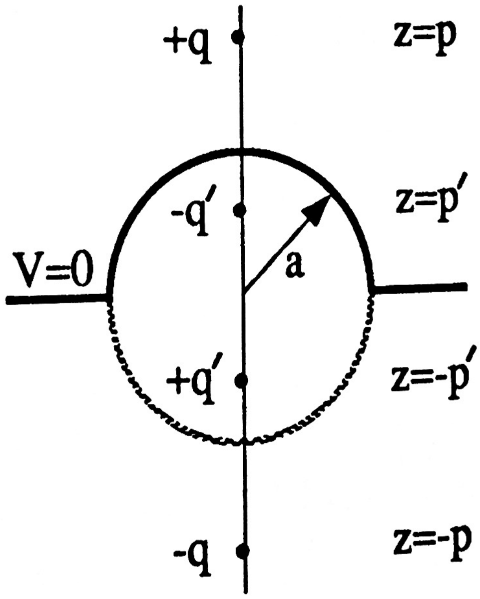


Figure 11.1.

We recall the rules of the image charge game: we want to replace the conductor with a distribution of "virtual charges" in such a way that the boundary conditions are satisfied. (See Jackson, Chapter 2.) If we ignore the plane for a moment and concentrate on the hemispherical bulge, it is easy to check that a second charge $-q'$, with $q' = qa/p$, at the point $z = p' = a^2/p$ leaves the surface of the bulge at a constant potential $V = 0$. This is the standard solution to the problem of a charge outside a spherical conductor. Now the trick is to introduce two more image charges: one of charge $-q$ at point $z = -p$, and one of charge q' at point $z = -p'$. These two charges do not change the potential on the hemispherical bulge, but now the potential also vanishes everwhere on the infinite plane.

The beauty of image-charge techniques now becomes apparant. The original charge feels the same force from a conducting infinite plane with a hemispherical bulge as it would from a set of three charges with the magnitudes and positions given above. So the force in the z-direction is given by

$$F = \sum_{q_i} \frac{qq_i}{r^2}, \tag{11.1}$$

where the sum is over the set of image charges. Our final answer is

$$\begin{aligned} F &= \frac{-qq'}{(p-p')^2} + \frac{qq'}{(p+p')^2} - \frac{q^2}{(2p)^2} \\ &= -q^2\left(\frac{ap}{(p^2-a^2)^2} - \frac{ap}{(p^2+a^2)^2} + \frac{1}{4p^2}\right). \end{aligned} \tag{11.2}$$

Solution 2.5. We wish to find the angular velocity of the wheel $\omega(t)$ and the current in the circuit $I(t)$. First we define the current to be positive if it flows out of the positive terminal of the battery. Let the origin of our coordinate system be at the hub of the wheel and $\hat{\mathbf{z}}$ be parallel to $\mathbf{B}$. Then if $\mathbf{r}$ gives the position along the current-carrying spoke of the wheel, the force on an infinitesimal element of this arm is

$$d\mathbf{F} = \frac{I}{c} d\mathbf{r} \times \mathbf{B}. \tag{11.52}$$

The torque on the wheel is therefore

$$J\dot{\omega}\,\hat{\mathbf{z}} = \int \mathbf{r} \times d\mathbf{F} = \int_0^R \frac{IB}{c} r\,dr\,\hat{\mathbf{z}}, \tag{11.53}$$

which leads to

$$J\dot{\omega}\hat{\mathbf{z}} = \frac{IBR^2}{2c}\hat{\mathbf{z}}. \tag{11.54}$$

We now have one equation relating $\omega(t)$ and $I(t)$. To find a second equation, we set the power delivered by the battery equal to the power absorbed by the rest of the circuit:

$$IV = I\left(L\dot{I}\right) + \frac{d}{dt}\left(\frac{1}{2}J\omega^2\right), \tag{11.55}$$

or

$$I\left(V - L\dot{I}\right) = J\omega\dot{\omega}. \tag{11.56}$$

Differentiating the torque equation (11.54) gives

$$\dot{I} = J\ddot{\omega}\left(\frac{2c}{BR^2}\right). \tag{11.57}$$

Substituting this expression into equation (11.56) and using (11.54) to cancel a common factor of I leads to a second order differential equation for $\omega(t)$:

$$\ddot{\omega} + \left(\frac{R^2B}{2c}\right)^2 \frac{1}{LJ}\omega = \frac{R^2B}{2cLJ}V. \tag{11.58}$$

This equation has the general solution

$$\omega(t) = C\cos\Omega t + D\sin\Omega t + \frac{2cV}{BR^2}, \tag{11.59}$$

where C and D are constants to be determined from the initial conditions and

$$\Omega = \frac{R^2B}{2c\sqrt{LJ}}. \tag{11.60}$$

To find the coefficient D we note that at $t = 0$ there is no current. Because $I(0) = 0$ and because $\dot{\omega}$ is proportional to $I(t)$ from equation (11.54), we have $\dot{\omega}(0) = 0$ and thus $D = 0$. To find C we note that at $t = 0$ the wheel is at rest, and thus $\omega(0) = 0$. Therefore $C = -2cV/BR^2$, and the final solution for $\omega(t)$ is

$$\omega(t) = \frac{2cV}{BR^2}(1 - \cos\Omega t). \tag{11.61}$$

We then use equation (11.54) to find

$$I(t) = \frac{V}{\Omega L}\sin\Omega t. \tag{11.62}$$

This problem can also be solved by the application of Kirchoff's law instead of the conservation of energy equation (11.55). When setting the sum of the voltage changes around the circuit equal to zero (Kirchoff's law), one must include the voltage induced by the changing flux through the circuit.

[1.] An electron is bound to a spring with spring constant k.

(a) Calculate both the differential and total scattering cross sections for unpolarized EM waves incident on the electron.

SOLUTION: Without loss of generality, I can consider a plane wave with polarization in the $\hat{x}$ direction. The equation of motion for the electron in the plane wave is then:

$$m\dot{v}_x = -kx - eE_o \exp(-i\omega t)$$

We'll assume that we have time harmonic plane waves and that the response of the electron in the long time limit is at the same frequency of the wave (strictly speaking we'd need some damping here to kill of the initial transient at the resonant frequency, but we can consider this as the limit of very small damping coefficient). Then the solution for the position of the electron as a function of time is:

$$x = \frac{-eE_o \exp(-i\omega t)}{m(\omega_o^2 - \omega^2)}$$

We can take several approaches to get the cross section from this. One way is to recall the angular radiation pattern for electric dipole radiation:

$$\frac{dP}{d\Omega} = \frac{\mu_o}{16\pi^2 c} \left|\hat{r} \times \ddot{\mathbf{p}}_{\text{ret}}\right|^2$$

The dipole moment of our oscillating electron is $\mathbf{p} = -ex\hat{x}$

So

$$\frac{dP}{d\Omega} = \frac{\mu_o}{16\pi^2 c} \frac{e^4 E_o^2 \omega^4}{m^2(\omega_o^2 - \omega^2)^2} \left|\hat{r} \times \hat{x}\right|^2 \left|\exp(-i\omega t)\right|^2$$

Taking a time average and rearranging terms gives us:

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{r_e^2}{2} \frac{c\epsilon_o E_o^2 \omega^4}{\omega_o^2 - \omega^2} \left|\hat{r} \times \hat{x}\right|^2$$

Where

$$r_e = \frac{e^2}{4\pi\epsilon_o mc^2}$$

The differential cross section is just the angular distribution of radiation power divided by the incident pointing flux, $\langle|\mathbf{S}_{\text{inc}}|\rangle = c\epsilon_o E_0^2/2$

$$\left\langle \frac{d\sigma}{d\Omega} \right\rangle = \frac{\left\langle \frac{dP}{d\Omega} \right\rangle}{\langle|\mathbf{S}_{\text{inc}}|\rangle} = \frac{r_e^2\omega^4}{(\omega_o^2-\omega^2)^2}\,|\hat{r}\times\hat{x}|^2$$

This is for light polarized in the $\hat{x}$ direction. We re-write the angular term:

$$|\hat{r}\times\hat{x}|^2 = \left|\hat{k}\times\hat{e}_o\right|^2 = \left(1-\left|\hat{k}\cdot\hat{e}_o\right|^2\right)$$

Where $\mathbf{k}$ is the wavenumber of the emitted/scattered radiation (along $\hat{r}$) and $\hat{e}_o$ is the polarization vector of the incident wave (which has wavenumber $\mathbf{k}_o$. We'll argue that, on average, unpolarized light contains equal amounts of light with polarization in the plane defined by $\mathbf{k}$ and $\mathbf{k}_o$ ($\hat{e}_{\|}$) and light with polarization perpendicular to this plane ($\hat{e}_{\perp}$). Then the total unpolarized differential cross section is

$$\begin{aligned}\left\langle \frac{d\sigma}{d\Omega} \right\rangle_{\text{unpol}} &= \frac{r_e^2\omega^4}{(\omega_o^2-\omega^2)^2}\left(\frac{1}{2}(1-\left|\hat{k}\cdot\hat{e}_{\|}\right|^2)+\frac{1}{2}(1-\left|\hat{k}\cdot\hat{e}_{\perp}\right|^2)\right)\\ &= \frac{r_e^2\omega^4}{(\omega_o^2-\omega^2)^2}\frac{1}{2}(1+\cos^2\theta)\end{aligned}$$

Where θ is the angle between $\hat{k}$. The total cross section is then just

$$\begin{aligned}\langle\sigma\rangle_{\text{unpol}} &= \int d\Omega \left\langle \frac{d\sigma}{d\Omega} \right\rangle_{\text{unpol}}\\ &= \int \sin\theta d\theta d\phi \frac{r_e^2\omega^4}{(\omega_o^2-\omega^2)^2}\frac{1}{2}(1+\cos^2\theta)\\ &= \frac{8\pi}{3}r_e^2\frac{\omega^4}{(\omega_o^2-\omega^2)^2}\end{aligned}$$

(b) In what limit should the total cross section equal the Thomson scattering cross section? Take this limit and confirm that it results in the Thomson scattering cross section.

SOLUTION: The correct limit is $\omega \gg \omega_o$, where ω_o is the resonant frequency of the electron-spring system. In this limit, the electron behaves like a free particle. Taking this limit in our expression above yields:

$$\langle\sigma\rangle_{\text{unpol}} \xrightarrow[\omega\gg\omega_o]{} \frac{8\pi}{3}r_e^2$$

Which is the Thomson scattering cross-section.

(c) In what limit should the total cross section yield Rayleigh scattering? Take this limit and confirm that the cross section is consistent with Rayleigh scattering (what is the frequency dependence you expect?)

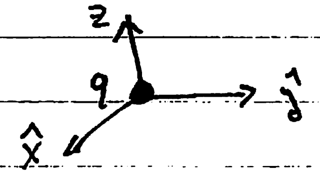
SOLUTION: The correct limit is $\omega \ll \omega_o$, where ω_o is the resonant frequency of the electron-spring system. In this limit, you find an ω^4 dependence in the scattering cross-section (leading to the blue sky explanation):

$$\langle \sigma \rangle_{\text{unpol}} \xrightarrow[\omega \ll \omega_o]{} \frac{8\pi}{3} r_e^2 \frac{\omega^4}{\omega_o^4}$$

Q 14.

E+M #1 (answer).

a)



$\nabla\cdot\underline{E} = 4\pi(\rho_{ext} + \rho_p)$ is Poisson's Eq

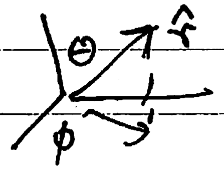
where $\rho_{ext} = q\,\delta(\underline{r})$; ρ_p is induced polarization chg

using displacement vector $\underline{D} = \overleftrightarrow{\epsilon}\cdot\underline{E}$ transforms Poisson's Eq into: $\nabla\cdot\underline{D} = 4\pi q\,\delta(\underline{r})$

solvable by Gauss' law to: $\underline{D} = \dfrac{q}{r^2}\hat{r}$

with $r = (x^2+y^2+z^2)$; $\hat{r} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}$

in spherical coordinates



or $\hat{r} = \dfrac{x}{r}\hat{x} + \dfrac{y}{r}\hat{y} + \dfrac{z}{r}\hat{z}$

$$\Rightarrow \quad \underline{D} = \frac{q}{r^3}\left[x\hat{x} + y\hat{y} + z\hat{z}\right]$$

$$\& \quad \overleftrightarrow{\epsilon}\cdot\underline{E} = \epsilon_\perp E_x\hat{x} + \epsilon_\perp E_y\hat{y} + \epsilon_{\parallel} E_z\hat{z}$$

$$\Rightarrow \quad E_x = \frac{q}{r^3\epsilon_\perp}x, \quad E_y = \frac{q}{r^3\epsilon_\perp}y, \quad E_z = \frac{q}{r^3\epsilon_{\parallel}}z$$

& the magnitude is: $|\underline{E}| = \sqrt{E_x^2 + E_y^2 + E_z^2}$

$$\boxed{|\underline{E}| = \frac{q}{r^3}\left[\frac{x^2+y^2}{\epsilon_\perp^2} + \frac{z^2}{\epsilon_{\parallel}^2}\right]^{1/2}}$$

E+M #1 (answer/cont..)

b) For $r \neq 0$ all the charge density is due to polarization of the dielectric, i.e., $\nabla \cdot \underline{E} = 4\pi \rho_p \Rightarrow \rho_p = \frac{1}{4\pi} \nabla \cdot \underline{E}$

but $\nabla \cdot \underline{E} = \partial_x E_x + \partial_y E_y + \partial_z E_z$

and $\partial_x E_x = \frac{q}{\epsilon_\perp} \frac{\partial}{\partial x}\left(\frac{x}{r^3}\right) = \frac{q}{\epsilon_\perp}\left[\frac{1}{r^3} - \frac{3x^2}{r^5}\right]$

$\partial_y E_y = \frac{q}{\epsilon_\perp} \frac{\partial}{\partial y}\left(\frac{y}{r^3}\right) = \frac{q}{\epsilon_\perp}\left[\frac{1}{r^3} - \frac{3y^2}{r^5}\right]$

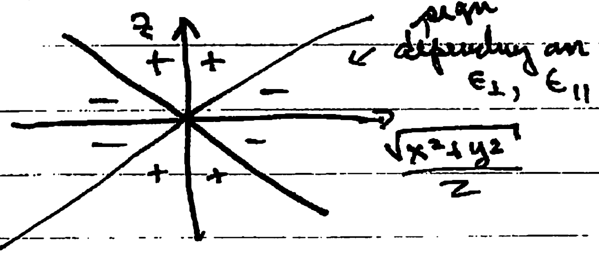
$\partial_z E_z = \frac{q}{\epsilon_\parallel} \frac{\partial}{\partial z}\left(\frac{z}{r^3}\right) = \frac{q}{\epsilon_\perp}\left[\frac{1}{r^3} - \frac{3z^2}{r^5}\right]$

$$\Rightarrow \nabla \cdot \underline{E} = \frac{q}{r^5}\left\{\frac{(2z^2 - x^2 - y^2)}{\epsilon_\perp} + \frac{x^2 + y^2 - 2z^2}{\epsilon_\parallel}\right\}$$

$$\nabla \cdot \underline{E} = \frac{q}{r^5}\left[2z^2 - (x^2+y^2)\right]\left(\frac{1}{\epsilon_\perp} - \frac{1}{\epsilon_\parallel}\right)$$

$$\Rightarrow \boxed{\rho_p(x,y,z) = \frac{q}{4\pi r^5}\left[2z^2 - (x^2+y^2)\right]\left(\frac{1}{\epsilon_\perp} - \frac{1}{\epsilon_\parallel}\right)}$$

shows charge cones



c) Total electrical energy density is $u_E = \frac{1}{2}\underline{E} \cdot \underline{D} = \frac{1}{2}\underline{E} \cdot \overleftrightarrow{\epsilon} \cdot \underline{E}$

but $\overleftrightarrow{\epsilon} \cdot \underline{E} = \epsilon_\perp E_x \hat{x} + \epsilon_\perp E_y \hat{y} + \epsilon_\parallel E_z \hat{z}$

& $\underline{E} \cdot \overleftrightarrow{\epsilon} \cdot \underline{E} = \epsilon_\perp E_x^2 + \epsilon_\perp E_y^2 + \epsilon_\parallel E_z^2$

Q14, p3

IV.

E+M #1 (answer) (cont-)

$$u_E = \frac{1}{2}\left[\epsilon_\perp \frac{q^2}{r^6\epsilon_\perp^2}x^2 + \frac{q^2}{r^6\epsilon_\perp^2}y^2 + \frac{q^2}{r^6\epsilon_\parallel^2}z^2\right]$$

$$\boxed{u_E = \frac{q^2}{2r^6}\left[\frac{x^2+y^2}{\epsilon_\perp} + \frac{z^2}{\epsilon_\parallel}\right]}$$