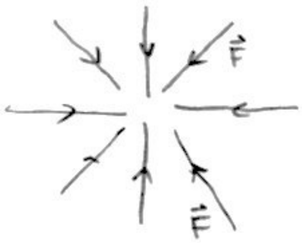
Fall 2011 Q14:

a) Prove that it is impossible for (q, m) to be held in a stable equilibrium by some arrangement of static external charges (Earnshaw's Theorem).

b) Show a stable equilibrium in $\vec{B}(\vec{x}), -g\hat{z}$ fields exists for a diamagnetic sphere of radius 'a' and $\vec{m}$ given by:

$$\vec{m} = -\underbrace{\frac{|\chi| \frac{4}{3}\pi a^3}{\mu_0}}_{m_0} \vec{B}(\vec{x})$$

a) For equilibrium, need a negative divergence of the force field:



$$-\vec{\nabla}.\vec{F} > 0$$

$$\vec{F} = -\vec{\nabla} V(\vec{x})$$

i.e. need local minimum of $V(\vec{x})$: $-\vec{\nabla}.\vec{F} = \nabla^2 V(\vec{x}) > 0$

$$\nabla^2 V(\vec{x}) = m \nabla^2 \Phi_g + q \nabla^2 \Phi_e > 0$$

However, here we have some arrangement of <u>external charges</u>, so

$$m \nabla^2 \Phi_g + q \nabla^2 \Phi_e = 0$$

Therefore we can only create a point of inflection in the force field, e.g. a saddle point, but not a stable equilibrium.

b)

$$V = -\vec{m}.\vec{B}(\vec{x}) = m_0 \vec{B}.\vec{B} = m_0 |B(\vec{x})|^2$$

$\nabla^2 V(\vec{x}) = m_0 \nabla^2 |B(\vec{x})|^2 > 0$ for any spatially varying field, e.g. $\vec{B} = B_0 x \hat{x}$

$|B|^2 = B_0^2 x^2$

$\nabla^2 x^2 > 0$