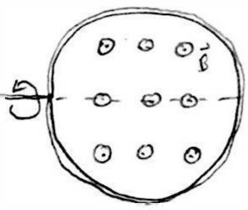
1. **Previous Midterm.**

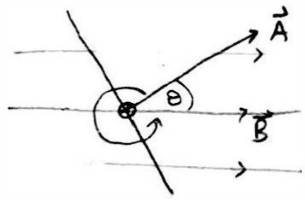
(a) A motor with a brush-and-commutator arrangement has a circular coil with radius 2.5cm and 150 turns of wire. The magnetic field has magnitude 0.060T and the coil rotates at 440 (rev/min.). The arrangement is akin to a tennis racket rotating on an axis along its handle in a magnetic field perpendicular to that axis. What is the maximum emf induced in the coil?



$$\mathcal{E} = -N \frac{d\Phi_B}{dt}, \quad \text{where } \mathcal{E} = \text{induced voltage}$$

N = # of turns

$\Phi_B = \vec{B}\cdot\vec{A} = BA\cos\theta$



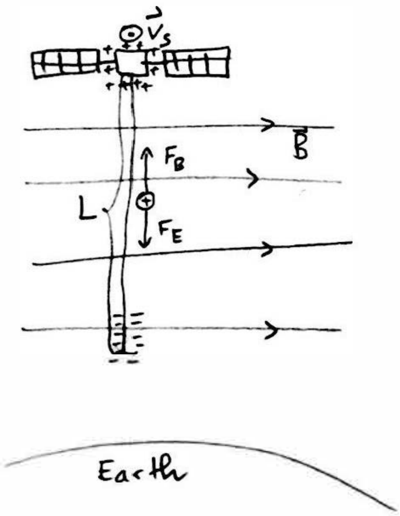
Find $\theta(t) = \omega t = 2\pi f t = \frac{2\pi t}{T}$

$$\Rightarrow \frac{d\Phi_B}{dt} = BA\frac{d(\cos\omega t)}{dt} = -BA\omega\sin\omega t$$

$$\Rightarrow \mathcal{E}(t) = NBA\omega\sin(\omega t), \quad \max\{\sin(\omega t)\} = 1$$

$$\Rightarrow \mathcal{E}_{max} = NBA\omega = \underline{0.814\,V}$$

(b) Some scientists propose to provide power for a satellite by dropping a long conducting tether. A lightweight 100m long conducting wire is unspooled from the satellite and hangs perpendicular to the earth's surface. Assume the earth's magnetic field is essentially horizontal and has a value of $8 \times 10^{-5}T$ at the altitude of the satellite; ignore any changes in B with altitude. The satellite is travelling about 420 km/minute. What is the voltage generated by the tether? List one physical consequence of this arrangement after many orbits of the satellite.



$F_B = F_E$ (in equilibrium)

$$|\vec{F}_B| = |q\vec{v}_s \times \vec{B}| = qv_sB = F_E = qE$$

$$\oint \vec{E}\cdot d\vec{l} = V$$

$$\Rightarrow E = \frac{V}{L}$$

$$\Rightarrow F_B = F_E = qv_sB = \frac{qV}{L}$$

$$\Rightarrow \boxed{V = v_sBL = 56V}$$

Physical Consequence: After charge build-up, no more current flows (open circuit). Charge-up might harm electronics. ⇒ Still need solar panels!

2. **YF 27.19**. *Nuclear Fusion.* If two deuterium nuclei get close enough together, the attraction of the strong nuclear force will fuse them to make an isotope of helium, releasing vast amounts of energy. This is the principle behind the fusion reactor. The deuterium gas is much too hot to be contained by material walls, so they are confined in a "magnetic bottle". **Useful constant**: $\epsilon_0 = 8.85 \times 10^{-12} F/m$.
How fast would two nuclei have to move so that in a head-on collision they would get close enough to fuse? Treat the nuclei as point charges +e and mass 3.34×10^{-27}kg, and assume that a separation of 1.0×10^{-15}m is needed for fusion.

a) Energy conservation: $E_i = E_f$

initial: $E_i = \cancel{PE} + KE = 2 \cdot (\tfrac{1}{2} m v^2)$

final: $E_f = PE + \cancel{KE} = \Delta PE = PE(r_0) - \cancel{PE(\infty)} = \dfrac{e^2}{4\pi \varepsilon_0 r_0}$

$$E_i = E_f = m v^2 = \frac{e^2}{4\pi \varepsilon_0 r_0}$$

$$\Rightarrow \boxed{v = \frac{e}{(4\pi \varepsilon_0 r_0 m)^{1/2}} = 8 \times 10^6 \ m/s}$$

$\hookrightarrow v \ll c \Rightarrow$ non-relativistic.