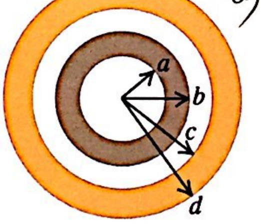
Name: 1B Discussion - Week 6

1. **Electric force within the Nucleus** *(YF 13th ed. 21.80)*. Typical dimensions of atomic nuclei are of the order of $10^{-15}m$ ($1fm$). (a) If two protons in a nucleus are $2.0fm$ apart, find the magnitude of the electric force each one exerts on the other. Express the answer in newtons [redacted]. Would this force be large enough for a person to feel? (b) Since the protons repel each other so strongly, why don't they shoot out of the nucleus?

a) 2fm $\quad F = \dfrac{k_e q^2}{r^2} = \dfrac{9\times10^9\ Nm^2/C^2 \times (1.6\times10^{-19})^2}{2\times10^{-15}} = 58N$

b) Something must be holding the nucleus together by opposing this enormous repulsion. This is the strong nuclear force.

2. **Concentric Spherical Shells** *(YF 13th ed. 22.47)*. In the figure below, the inner shell has total charge $+2q$, and the outer shell has charge $+4q$. (a) Calculate the electric field (magnitude and direction) in terms of q and the distance r from the common center of the two shells for (i) $r < a$; (ii) $a < r < b$; (iii) $b < r < c$; (iv) $c < r < d$; (v) $r > d$. Show your results in a graph of the radial component of $\vec{\mathbf{E}}$ as a function of r. (b) What is the total charge on the (i) inner surface of the small shell; (ii) outer surface of the small shell; (iii) inner surface of the large shell; (iv) outer surface of the large shell?



a) Apply Gauss' law: $E\,4\pi r^2 = \dfrac{Q_{encl.}}{\varepsilon_0}$

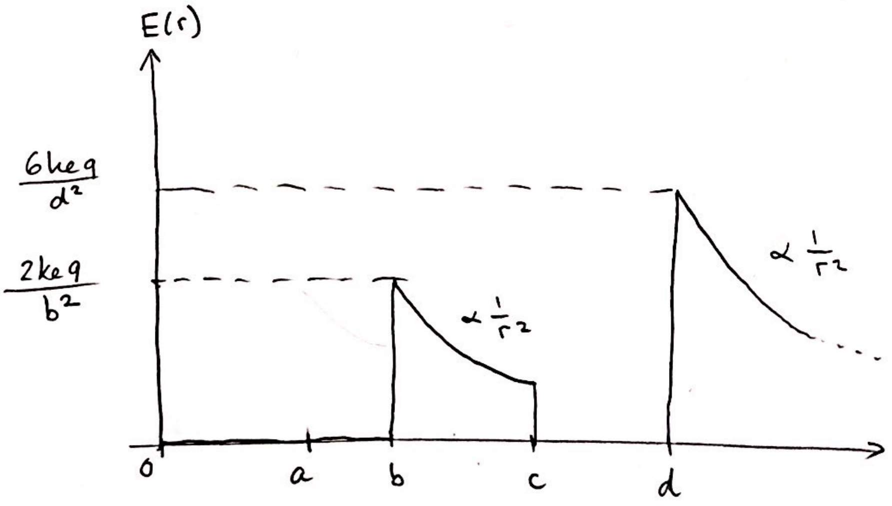
$r<a$: $E = 0$

$a<r<b$: $E = 0$ in conductor

$b<r<c$: $E = \dfrac{k_e 2q}{r^2}\hat{r}$ (radially out by symmetry)

$c<r<d$: $E = 0$

$d<r$: $E = \dfrac{6k_e q}{r^2}\hat{r}$



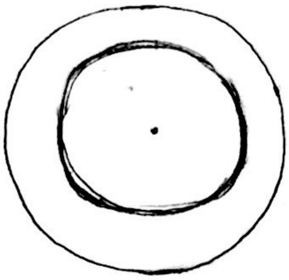
b) (i) $Q_{encl} = 0$, $q_s = 0$

(ii) $Q_{encl} = 2q$, $q = 2q$

(iii) $Q_{encl} = 0$, $q = -2q$

(iv) $Q_{encl} = 6q$, $q = 6q$

3. **Self-Energy of a Sphere of Charge** *(YF 13th ed. 23.71)*. A solid sphere of radius R contains a total charge Q distributed uniformly throughout its volume. Find the energy needed to assemble this charge by bringing infinitesimal charges from far away. This energy is called the "self-energy" of the charge distribution. (Hint: After you have assembled a charge q in a sphere of radius r how much energy would it take to add a spherical shell of thickness dr having charge dq? Then integrate to get the total energy.)



$$\rho = \frac{Q}{\text{Vol.}} = \text{const.} = \frac{3Q}{4\pi R^3} \qquad q(r) = \rho \frac{4}{3}\pi r^3$$

$$dq = \rho 4\pi r^2 dr$$

$$dU = V(r)\, dq \quad , \text{where } V(r) = \frac{k_e q(r)}{r}$$

$$U = \int_\infty^R V(r)\, dq = \int_\infty^R \frac{k_e q(r)}{r} \rho 4\pi r^2 dr =$$

$$= \int_\infty^R k_e \rho^2 \frac{4}{3}\pi r^3 4\pi r\, dr =$$

$$= \frac{k_e \rho^2 (4\pi)^2}{5 \times 3} R^5 =$$

$$\rho = \frac{3Q}{4\pi R^3}$$

$$= \frac{3}{5}\left(\frac{1}{4\pi\varepsilon_0} \frac{Q^2}{R}\right)$$